Math 420, Spring 2022
Project One: Metrics

This explores whether the metrics assigned to your team are leading market indicators.

(1) Describe each of the nine assigned assets assigned on the "Projects" page including the holdings of each fund and the business model of each company. (This information should inform some of your subsequent answers.)

(2) Use adjusted closing prices to compute the return for each trading day over the years 2006-2021. For each the years ending 31 December 2006-2021 use one-year histories and uniform weights to compute $\mathbf{m}$ and $\mathbf{V}$ for the assets in
- group (A),
- group (AB) (= groups (A) and (B) combined),
- group (ABC) (= groups (A), (B), and (C) combined).

Compute the Markowitz frontier parameters (σ_{mv}, μ_{mv}, and ν_{mv}) for each $\mathbf{m}$ and $\mathbf{V}$. Present these parameters in three tables (one for each of the groups (A), (AB) and (ABC)) with one column for each year (2006-2021) that lists the parameter values rounded to the nearest thousandth. Comment on the implications of what you find.

(3) Compute the minimum volatility allocation $\mathbf{f}_{\text{mv}}$ for each $\mathbf{m}$ and $\mathbf{V}$ computed in Part 2. For each minimum volatility portfolio compute its leverage and liquidity metrics. Present these in three tables (one for each of the groups (A), (AB) and (ABC)) with one column for each year (2006-2021) that lists the allocation for each asset and the metric values for the portfolio rounded to the nearest thousandth. Comment on the implications of what you find.

(4) Let μ_{si} and μ_{cl} be the risk-free rates computed from the U.S. Treasury Bill rate on the last trading day of each of the years 2006-2021 as discussed in the slides. For each of these rates compute the Tobin frontier parameter and the tangency point and asymptote intersection metrics for each $\mathbf{m}$ and $\mathbf{V}$ computed in Part 2. Present these in three tables (one for each of the groups (A), (AB) and (ABC)) with one column for each year (2006-2021) that lists μ_{si}, ν_{si}, $\omega_{\text{si}}^{\text{tg}}$, $\omega_{\text{si}}^{\text{as}}$, μ_{cl}, ν_{cl}, $\omega_{\text{cl}}^{\text{tg}}$, and $\omega_{\text{cl}}^{\text{as}}$ rounded to the nearest thousandth. Comment on the implications of what you find.

(5) For each asset in group (A) compute the liquidity and Sharpe metrics for each year (2006-2021) and the efficiency and proximity metrics relative to the (ABC) Markowitz frontier whose parameters where computed in Part 2. Present these in a table with one column for each year (2006-2021) that lists ω_i^{δ}, ω_i^{ρ}, ω_i^{μ} and ω_i^{σ} for each of the three assets ($i = 1, 2, 3$) rounded to the nearest thousandth. Comment on the implications of what you find.

(6) Compute the safe tangent allocation $\mathbf{f}_{\text{st}}$ for each $\mathbf{m}$ and $\mathbf{V}$ computed in Part 2 for which $\mu_{\text{si}} \neq \mu_{\text{mv}}$. For each safe tangent portfolio that exists compute its leverage, liquidity, and Sharpe metrics. Present these in three tables (one for each of the groups (A), (AB) and (ABC)) with one column for each year (2006-2021) that lists the allocation for each asset and the metrics for the portfolio rounded to the nearest thousandth. Comment on the implications of what you find.

(7) Compute the credit tangent allocation $\mathbf{f}_{\rm ct}$ for each $\mathbf{m}$ and $\mathbf{V}$ computed in Part 2 for which $\mu_{\rm cl} \neq \mu_{\rm mv}$. For each credit tangent portfolio that exists compute its leverage, liquidity, and Sharpe metrics. Present these in three tables (one for each of the groups (A), (AB) and (ABC)) with one column for each year (2006-2021) that lists the allocation for each asset and the metrics for the portfolio rounded to the nearest thousandth. Comment on the implications of what you find.

(8) Compute the efficient long tangent allocation $\mathbf{f}_{\rm elt}$ for each $\mathbf{m}$ and $\mathbf{V}$ computed in Part 2. For each efficient long tangent portfolio compute its liquidity, efficiency, proximity, and Sharpe metrics. Present these in three tables (one for each of the groups (A), (AB) and (ABC)) with one column for each year (2006-2021) that lists $\mu_{\rm si}$, $\mu_{\rm mn}$, $\mu_{\rm mx}$, $\sigma_{\rm mx}$, $\sigma_{\rm elf}$, $\mu_{\rm elf}$, the allocation for each asset and the metrics for the portfolio rounded to the nearest thousandth. Comment on the implications of what you find.

(9) For each year (2006-2021) use the $\mathbf{m}$ and $\mathbf{V}$ computed in Part 2 for group (ABC) and the $\mu_{\rm si}$ and $\mu_{\rm cl}$ from Part 4 to give a graph in the $\sigma\mu$-plane that includes the following.

- the volatility and return mean of each asset;
- the Markowitz frontier and asymptotes;
- the points $(0, \mu_{\rm si})$, $(0, \mu_{\rm cl})$, and $(0, \mu_{\rm mv})$;
- the points $(\sigma_{\rm st}, \mu_{\rm st})$ and $(\sigma_{\rm ct}, \mu_{\rm ct})$ whenever they exist;
- the efficient frontier for the Two-Rate model;
- the long frontier;
- the efficient long frontier with the safe investment and the point $(\sigma_{\rm elt}, \mu_{\rm elt})$;
- the limited-leverage frontiers for $\ell = 1$ and $\ell = 5$;
- the efficient limited-leverage frontiers for $\ell = 1$ and $\ell = 5$ with Two-Rates.

There should be at least 16 graphs, one for each year. Making comparisons is easier if they all have the same axis scales. However, a few extreme years may make this impractical if the remaining graphs become underresolved. In that case use one axis scale for most years and display the extreme years on an appropriate scale. Comment on the implications of what you see.

(10) (optional movie) Rather than showing sixteen $\sigma\mu$-plane graphs during your oral presentation, you can show the evolution as a movie. You can make the move better by generating more than 16 frames. This can be done by using rolling one-year averages to compute a new $\mathbf{m}$ and $\mathbf{V}$ either every quarter (61 frames) or every month (181 frames). This increased resolution requires downloading more U.S. Teasury Note data, but no more risky asset data. Movies can help you see an implication of the data that you might not see other wise.

(11) Explore if any of the three classes of metrics assigned to your team warn of a potential economic downturn by comparing them with VFINX. For each class of metrics, plot them as a fuction of year in the ωt-plane. Try to identify behaviors that always or almost always preceed a downturn. (The does not mean that a downturn must always follow these behaviors!)

(12) (optional higher resolution) Rather than plotting each class of metrics as a function of year, the rolling one-year averages idea described in the movie option can be used to plot each class of metrics as a function of quarters (61 one-year avearges) or months (181 one-year averages). This can be done without doing the movie option!