Math 420, Spring 2022
Last Assignment

3:00pm Saturday, 14 May, 2022

In the following exercises consider the risky assets in groups (A), (B) and (C) assigned on the class website under **Project Two**. Use adjusted closing prices to compute the return of each asset for each trading day over the last sixteen calendar years — namely, the years ending on December 31 of 2006-2021.

There are 64 quarters within this sixteen year period. There are 61 one-year periods within these sixteen years — the first consisting of quarters 1-4, the second consisting of quarters 2-5, and so on until the last consisting of quarters 61-64. We call the return histories over these 61 one-year periods *rolling histories* and label each by its first quarter.

For each of the 61 one-year periods compute $\mathbf{m}$ and $\mathbf{V}$ using uniform weights for the assests in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined. Use the U.S. T-Bill (13 week) rate available at the last trading day of each history as the safe investment rate for the data from that period. Assume that the credit line rate for each period is three points higher than the U.S. T-Bill rate. For each of the 61 one-year periods and each of the three pairs $(\mathbf{m}, \mathbf{V})$ compute

- the allocation $\mathbf{f}_{\mathrm{st}}$ for the safe tangent portfolio on the Markowitz frontier,
- the allocation $\mathbf{f}_{\mathrm{ct}}$ for the cedit tangent portfolio on the Markowitz frontier,
- the allocation $\mathbf{f}_{\mathrm{elt}}$ for efficient long tangent portfolio on the long frontier.

There are a total on nine tangent portfolios.

Task 1. For each of the nine assets and each of the nine tangent portfolios use uniform weights to compute the signal-to-noise metrics $\omega^{\hat{\mu}}$ and $\omega^{\hat{\xi}}$ for each of the 61 one-year periods. Here

$$\omega^{\hat{\mu}} = \frac{1}{1 + \mathrm{SNR}(\mu)^2}\,, \qquad \omega^{\hat{\xi}} = \frac{1}{1 + \mathrm{SNR}(\xi)^2}\,,$$

where $\mathrm{SNR}(\mu)$ and $\mathrm{SNR}(\xi)$ are the signal-to-noise ratios

$$\mathrm{SNR}(\mu) = \frac{\hat{\mu}}{\widehat{\mathrm{SD}}(\hat{\mu})} = \frac{\widehat{\mathrm{Ex}}(R)}{\widehat{\mathrm{SD}}\Big(\widehat{\mathrm{Ex}}(R)\Big)}\,, \qquad \mathrm{SNR}(\xi) = \frac{\hat{\xi}}{\widehat{\mathrm{SD}}(\hat{\xi})} = \frac{\widehat{\mathrm{Vr}}(R)}{\widehat{\mathrm{SD}}\Big(\widehat{\mathrm{Vr}}(R)\Big)}\,.$$

(Use the estimators from the example at the end of Section 6.3.) For each asset and each tangent portfolio plot these metrics as a function of the first quarter of each one-year period. There should be eighteen plots. How do the metrics for $\hat{\mu}$ comapre with those for $\hat{\xi}$? How do the metrics for the tangent portfolios compare with those of the individual assets? For which of these are you most certain of the values of $\hat{\mu}$ and $\hat{\xi}$? Give your reasoning.

Task 2. For each of the nine assets and each of the nine tangent portfolios use uniform weights to compute the metrics ω^{m}, ω^{v}, ω^{KS} and (optional) ω^{Ku} for each of the 61 one-year periods by comparing each quarter to every other quarter in the one-year period. (So that there are six comparisons made in each one-year period.) For each asset and each tangent portfolio plot these metrics as a function of the first quarter of each period. There should be eighteen plots. Which of the metrics seems most informative? How do the metrics for the tangent portfolios compare with those of the individual assets? For which of these are you most certain about the assumption of identically distributed? Give your reasoning.

Task 3. For each of the nine assets and each of the nine tangent portfolios use uniform weights to compute the metrics ω^{ar} and ω^{ac} for each of the 61 one-year periods. For each asset and each tangent portfolio plot these metrics as a function of the first quarter of each period. There should be eighteen plots. Which of the metrics seems most informative? How do the metrics for the tangent portfolios compare with those of the individual assets? For which of these are you most certain about the assumption of independent? Give your reasoning.

Task 4. Consider the family of reasonable estimators of the cautious objective

$$\widehat{\Gamma}_{\mathrm{r}}^{\chi} = \log(1+\hat{\mu}) - \tfrac{1}{2}\hat{\sigma}^2 - \chi\,\hat{\sigma}\,,$$

with $\chi = 0.000$, 0.0625, 0.1250, 0.1875, 0.2500, 0.3125, 0.3750, 0.4375 and 0.5000. For each of the first 57 one-year periods and each of these nine values of χ compute the maximizer of $\widehat{\Gamma}_{\mathrm{r}}^{\chi}$ over the set Λ_+ of long portfolios built from your nine risky assets and the safe investment. For each of these 57 one-year periods determine which of the nine maximizers performed best in the subsequent one-year period. Let χ_{opt} be the value of χ associated with this maximizer. Let ν_{lrf} be the Sharpe ratio of the efficient long tangent portfolio, which is defined by

$$\nu_{\mathrm{lrf}} = \begin{cases} \dfrac{\mu_{\mathrm{elt}} - \mu_{\mathrm{rf}}}{\sigma_{\mathrm{elt}}} & \text{if } \mu_{\mathrm{elt}} > \mu_{\mathrm{rf}}\,, \\ 0 & \text{otherwise}\,. \end{cases}$$

Plot χ_{opt} and ν_{lrf} as a function of the first quarter of each of these 57 one-year periods.

Task 5. Select four metrics from the first half of the course that you think should best inform your choice of caution coefficient χ. Give reasoning for your selection. Plot these metrics as a function of the first quarter of each of the 57 one-year periods from **Task 4**. Based on this plot and those from **Tasks 1-4** identify the metrics upon which the choice of caution coefficient should depend most strongly.