Math 420, Spring 2022
Solo Homework 2
Markowitz Frontiers

due Monday, 7 February, 2022

Consider the three groups of assets:

(A) VFINX, VBMFX, VGSLX;

(B) VIMAX, VSMAX, VGTSX;

(AB) VFINX, VBMFX, VGSLX, VIMAX, VSMAX, VGTSX.

Exercise 1. Use adjusted closing prices to compute the return for each trading day over the two years 2020-2021. Compute $\mathbf{m}$ and $\mathbf{V}$ for the assets in groups (A), (B), and (AB) using one-year histories with uniform weights for each the two years ending 31 December 2020 and 2021. Print out the three $\mathbf{m}$ and $\mathbf{V}$ for each of the two years 2020-2021 to four significant digits. What relationship do the three $\mathbf{m}$ and $\mathbf{V}$ for each year have to each other?

Exercise 2. For each $\mathbf{m}$ and $\mathbf{V}$ computed in Exercise 2, compute the minimum volatility portfolio allocation $\mathbf{f}_{\text{mv}}$. Present these in three tables (one for group (A), one for group (B), and one for group (AB)) that lists years and the allocations for each asset rounded to the nearest thousandth. Comment on how these change from year to year for the same groupings of assets. For each $\mathbf{f}_{\text{mv}}$ compute its leverage ratio $\lambda(\mathbf{f}_{\text{mv}})$ and its downside potential $\delta(\mathbf{f}_{\text{mv}})$.

Exercise 3. For each year graph in the $\sigma\mu$-plane:

- the volatility and return mean for that year of each asset;
- the three Markowitz frontiers associated with the appropriate $\mathbf{m}$ and $\mathbf{V}$ that were computed in Exercise 2;
- the volatility and return mean for that year of the three minimum volatility portfolios;
- the volatility and return mean for that year of the three Markowitz portfolios that are equidistributed in group (A), in group (B), and in group (AB).

There should be 2 graphs — one for each year — each with six assets, three frontiers, and six portfolios plotted. Use different symbols or colors to distinguish points associated with the different groups (A), (B), and (AB). Comment on any relationships that you see between the objects plotted on each graph. (This will be easier to do if you use the same scales for each of the graphs. Each σ-axis should begin at $\sigma = 0$.)

Exercise 4. Present a table that for each year gives the efficiency and proximity relative to the (AB) frontier to four significant digits for each asset that year. The table should have four columns of numbers (two for each year) and six rows (one for each asset). Comment on what these metrics say about the efficient market hypothesis.

Exercise 5. Present one graph for each year that plots the leverage ratio $\lambda(\mathbf{f}_{\text{f}}(\mu))$ as a function of μ for each of the three frontiers computed in Exercise 3. The μ-interval over which this is done should include each value of μ_{mv} and all minimizers of each $\lambda(\mathbf{f}_{\text{f}}(\mu))$ for that year. Comment on what these leverage ratio plots say about the market each year.