Math 420, Spring 2022
Team Homework 6

due Monday, 11 April, 2022

In the following exercises consider the risky assets in groups (A), (B) and (C) of your **Project Two**. Use adjusted closing prices to compute the return of each asset for each trading day over the last five calendar years — namely, the years ending on December 31 of 2017-2021.

There are 20 quarters within this five year period. There are 17 one-year periods within these five years — the first consisting of quarters 1-4, the second consisting of quarters 2-5, and so on until the last consisting of quarters 17-20. We call the return histories over these 17 one-year periods *rolling histories* and label each by its last quarter.

For each of the 17 one-year periods compute $\mathbf{m}$ and $\mathbf{V}$ using uniform weights for the assests in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined. Use the U.S. T-Bill (13 week) rate available at the beginning of each history as the safe investment rate for the data from that period. Assume that the credit line rate for each period is three points higher than the U.S. T-Bill rate.

Exercise 1. For each of the five calendar years plot a graph that shows

- the points (m_i, σ_i) associated with each asset,
- the frontier hyperbola associated with each of the three pairs $(\mathbf{m}, \mathbf{V})$,
- the points $(\mu_{\text{st}}, \sigma_{\text{st}})$ and $(\mu_{\text{ct}}, \sigma_{\text{ct}})$ on the frontier hyperbola associated with each of the three pairs $(\mathbf{m}, \mathbf{V})$,
- the efficient frontier associated with each of the three pairs $(\mathbf{m}, \mathbf{V})$,
- the long frontier associated with each of the three pairs $(\mathbf{m}, \mathbf{V})$,
- the point $(\mu_{\text{lt}}, \sigma_{\text{lt}})$ on the long frontier associated with each of the three pairs $(\mathbf{m}, \mathbf{V})$,
- the efficient long frontier associated with each of the three pairs $(\mathbf{m}, \mathbf{V})$.

Exercise 2. For each of the 17 one-year periods and each of the three pairs $(\mathbf{m}, \mathbf{V})$ compute

- the allocation $\mathbf{f}_{\text{st}}$ for the safe tangent portfolio on the Markowitz frontier,
- the allocation $\mathbf{f}_{\text{ct}}$ for the cedit tangent portfolio on the Markowitz frontier,
- the allocation $\mathbf{f}_{\text{elt}}$ for efficient long tangent portfolio on the long frontier.

Compute the signal-to-noise ratio of μ and ξ for the portfolios with allocations $\mathbf{f}_{\text{st}}$, $\mathbf{f}_{\text{ct}}$, and $\mathbf{f}_{\text{lt}}$ for each of the one-year periods and each of the three pairs $(\mathbf{m}, \mathbf{V})$. For each portfolio plot this ratio as a function of the last quarter of each period. There should be nine plots, one for each portfolio. How do these ratios compare with those of the individual assets? For which of these portfolios are you most certain of the expected value of its return? Give your reasoninng.

Exercise 3. Compute the metrics ω^{m}, ω^{v}, ω^{KS}, ω^{ar}, and ω^{ac} to the portfolios with allocations $\mathbf{f}_{\text{st}}$, $\mathbf{f}_{\text{ct}}$, and $\mathbf{f}_{\text{elt}}$ for each of the one-year periods and each of the three pairs $(\mathbf{m}, \mathbf{V})$. For ω^{m}, ω^{v}, and ω^{KS} compare each quarter with every other quarter in its one-year period, so there are six comparisons for each period. For each portfolio plot these five metrics as a function of the last quarter of each history. There should be nine plots, one for each portfolio. How do these metrics compare with those of the individual assets? Which of these portfolios is best described by an IID model? Give your reasoning.