Math 420, Spring 2022
Team Homework 4

due Friday, 1 April, 2022

In the following exercises consider the risky assets in groups (A), (B) and (C) of your **Project Two**. Use adjusted closing prices to compute the return of each asset for each trading day over the last five calendar years — namely, the years ending on December 31 of 2017-2021.

There are 20 quarters within this five year period. There are 17 one-year periods within these five years — the first consisting of quarters 1-4, the second consisting of quarters 2-5, and so on until the last consisting of quarters 17-20. We call the return histories over these 17 one-year periods *rolling histories* and label each by its last quarter.

Exercise 1. For each asset and each one-year period use uniform weights to compute $\hat{\mu} = \widehat{\mathrm{Ex}}(R)$ and the signal-to-noise ratio for $\mu = \mathrm{Ex}(R)$,

$$\mathrm{SNR}(\mu) = \frac{\hat{\mu}}{\widehat{\mathrm{SD}}(\hat{\mu})} = \frac{\widehat{\mathrm{Ex}}(R)}{\widehat{\mathrm{SD}}\Big(\widehat{\mathrm{Ex}}(R)\Big)}\,.$$

Present these results in two tables (one for $\hat{\mu}$ and one for $\mathrm{SNR}(\mu)$), each with 9 columns and 17 rows. Based on this, for each one-year period order the nine assets from that with the greatest certainity to least certainty in the estimate $\widehat{\mathrm{Ex}}(R)$. Can you explain the order?

Exercise 2. For each asset and each one-year period use uniform weights to compute $\hat{\gamma} = \widehat{\mathrm{Ex}}\big(\log(1+R)\big)$ and the signal-to-noise ratio for $\gamma = \mathrm{Ex}\big(\log(1+R)\big)$,

$$\mathrm{SNR}(\gamma) = \frac{\hat{\gamma}}{\widehat{\mathrm{SD}}(\hat{\gamma})} = \frac{\widehat{\mathrm{Ex}}\big(\log(1+R)\big)}{\widehat{\mathrm{SD}}\Big(\widehat{\mathrm{Ex}}\big(\log(1+R)\big)\Big)}\,.$$

Present these results in two tables (one for $\hat{\gamma}$ and one for $\mathrm{SNR}(\gamma)$), each with 9 columns and 17 rows. Based on this, for each one-year period order the nine assets from that with the greatest certainity to least certainty in the estimate $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$. (Here log is the natural logorithm.) Compare this ordering with the one that you found in Problem 1.

Exercise 3. Do you see a relation between $\hat{\mu}$ and $\hat{\gamma}$? Can you explain it? (Hint: Jensen.)

Exercise 4. For each asset and each one-year period use uniform weights to compute $\hat{\xi} = \widehat{\mathrm{Vr}}(R)$ and the signal-to-noise ratio for $\xi = \mathrm{Vr}(R)$,

$$\mathrm{SNR}(\xi) = \frac{\hat{\xi}}{\widehat{\mathrm{SD}}(\hat{\xi})} = \frac{\widehat{\mathrm{Vr}}(R)}{\widehat{\mathrm{SD}}\Big(\widehat{\mathrm{Vr}}(R)\Big)}\,.$$

(Use the estimators from the example at the end of Section 6.4.) Present these results in two tables (one for $\hat{\xi}$ and one for $\mathrm{SNR}(\xi)$), each with 9 columns and 17 rows. Based on this, for each year order the nine assets from that with the greatest certainity to least certainty in the estimate $\widehat{\mathrm{Vr}}(R)$. Can you explain the order? How do these signal-to-noise ratios compare with the ones that you found in Exercise 1?