Math 420, Spring 2022
Team Homework 3
Limited-Leverage Frontiers

due Monday, 28 February, 2022

Exercise 1. Use adjusted closing prices to computer the return for each trading day over the three years 2019-2021. For each the three years ending 31 December 2019, 2020, and 2021 use one-year histories and uniform weights to compute $\mathbf{m}$ and $\mathbf{V}$ for the assets in group (A), group (AB) (= groups (A) and (B) combined), group (ABC) (= groups (A), (B), and (C) combined). Print out the three $\mathbf{m}$ and $\mathbf{V}$ for each of the three years 2019-2021 to four significant digits. What relationship do the three $\mathbf{m}$ and $\mathbf{V}$ for each year have to each other?

Exercise 2. Let μ_{si} and μ_{cl} be the risk-free rates computed from the U.S. Treasury Bill rate on the last trading day of each of the years 2019-2021 as discussed in the slides.

- Compute the tangency point and asymptote intersection metrics for μ_{si} and μ_{cl} for each $\mathbf{m}$ and $\mathbf{V}$ computed in Exercise 1.

Present these metrics rounded to the nearest thousandth in two tables (one for each metric) with one column for each year (2019,2020,2021) and four rows (one for the risk-free rate and one for each group (A, AB, ABC)).

- Compute the tangent portfolio allocations $\mathbf{f}_{\text{st}}$ and $\mathbf{f}_{\text{ct}}$ associated with these risk-free rates whenever the risk-free rate does not equal μ_{mv} for the associated $\mathbf{m}$ and $\mathbf{V}$. For each tangent portfolio that exists compute its downside and Sharpe metrics.
- Compute the efficient long tangent portfolio allocation $\mathbf{f}_{\text{elt}}$ and its downside and Sharpe metrics.

Present these allocations and metrics in nine tables (three for group (A), three for group (AB), and three for group (ABC)) with one column for each year (2019,2020,2021) and one row for each asset and each metrics. Comment on how these change from year to year for the same groups of assets. For example: Do these tangent portfolios lie on the efficient or the inefficient Markowitz frontier?

Exercise 3. For each year and each group of assets graph in the $\sigma\mu$-plane:

- the volatility and return mean for that year of each asset;
- the Markowitz frontier and asymptotes for the $\mathbf{m}$ and $\mathbf{V}$ from Exercise 1;
- the long frontier for the $\mathbf{m}$ and $\mathbf{V}$ from Exercise 1;
- the volatility and return mean of each tangent portfolio from Exercise 2 for the $\mathbf{m}$ and $\mathbf{V}$ from Exercise 1;
- the efficient frontier and efficient long frontier for the $\mathbf{m}$ and $\mathbf{V}$ from Exercise 1 and the appropriate tangent portfolios from Exercise 2;
- the limited-leverage frontiers for leverage limit $\ell = 1$ and $\ell = 5$ for the $\mathbf{m}$ and $\mathbf{V}$ from Exercise 1.

There should be 9 graphs — one for each year and each group — each with the (σ_i, m_i) for each asset (either 3, 6, or 9), the Markowitz frontier and asymptotes, the long frontier, (σ, μ) for up to three tangent portfolios, the efficient frontier, the efficient long frontier, and the two limited-leverage frontiers. Comment on any relationships that you see between the plotted objects. (This will be easier to do if you use the same scales for each of the graphs. Each σ-axis should begin at $\sigma = 0$.) Each of these graphs should fill most of a page.