

# Math 420, Spring 2022

## Team Homework 1

### Frontiers with Risk-Free Assets

due Monday, 14 February, 2022

**Exercise 1.** Describe each asset in groups (A), (B), and (C) from your project. If it is a fund then describe the assets held by the fund. If it is a company then describe what it does. This information is easily found on the internet. Cite your sources. (A few sentences about each asset is all that you need.)

**Exercise 2.** Use adjusted closing prices to compute the return for each trading day over the three years 2019-2021. For each of the three years ending 31 December 2019, 2020, and 2021 use one-year histories and uniform weights to compute  $\mathbf{m}$  and  $\mathbf{V}$  for the assets in

- group (A),
- group (AB) = groups (A) and (B) combined,
- group (ABC) = groups (A), (B), and (C) combined.

Print out the three  $\mathbf{m}$  and  $\mathbf{V}$  for each of the three years 2019-2021 to four significant digits. What relationship do the three  $\mathbf{m}$  and  $\mathbf{V}$  for each year have to each other?

**Exercise 3.** For each  $\mathbf{m}$  and  $\mathbf{V}$  computed in Exercise 2 let  $\mu_{si}$  and  $\mu_{cl}$  be the risk-free rates computed from the U.S. Treasury Bill rate on the last trading day of each of the years 2019-2021 as discussed in the slides. Compute the tangent portfolio allocations  $\mathbf{f}_{st}$  and  $\mathbf{f}_{ct}$  associated with these risk-free rates whenever the risk-free rate does not equal  $\mu_{mv}$  for the associated  $\mathbf{m}$  and  $\mathbf{V}$ . Present these in six tables (two for group (A), two for group (AB), and two for group (ABC)) that lists years and the allocations for each asset rounded to the nearest thousandth. For each tangent portfolio that exists compute its downside potential, leverage, and stability ratio. Comment on how these change from year to year for the same groups of assets. For example: Do these tangent portfolios lie on the efficient or the inefficient Markowitz frontier?

**Exercise 4.** For each year graph in the  $\sigma\mu$ -plane:

- the three Markowitz frontiers associated with the three  $\mathbf{m}$  and  $\mathbf{V}$  from Exercise 2;
- the three efficient frontiers associated with the three  $\mathbf{m}$  and  $\mathbf{V}$  from Exercise 2 and the appropriate tangent portfolios from Exercise 3;
- the volatility and return mean for that year of each asset;
- the volatility and return mean of each tangent portfolio from Exercise 3.

There should be 3 graphs — one for each year — each with three Markowitz frontiers, three efficient frontiers, nine assets, and up to six tangent portfolios plotted. Use different symbols or colors to distinguish points associated with the different groups (A), (AB), and (ABC). Comment on any relationships that you see between the objects plotted on each graph. (This will be easier to do if you use the same scales for each of the graphs. Each  $\sigma$ -axis should begin at  $\sigma = 0$ .) What relationships do you see between the various Markowitz frontiers, efficient frontiers, and assets? Each of these graphs should fill most of a page.

**Exercise 5.** Present a table that for each year gives the Sharpe metric for each asset that year, all to four significant digits. The table should have three columns (one for each year) and nine rows (one for each year). Comment on what these metrics say.