Math 420, Spring 2021
Project One: Liquidity and Linkage

This project explores whether liquidity or linkage metrics are leading market indicators.

(1) Describe the nine assigned assets on the "Projects" page including the holdings of the funds. (This information should inform some of your subsequent answers.)

(2) For each of the years ending December 31 of 2006-2020 use one-year histories with uniform weights to compute $\mathbf{m}$ and $\mathbf{V}$ for the risky assets in group (A), in groups (A) and (B) combined, and in groups (A), (B), and (C) combined.

(3) For each $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 compute the minimum volatility portfolio allocation $\mathbf{f}_{\text{mv}}$. Present these in three tables (one for group (A), one for groups (A) and (B) combined, and one for groups (A), (B), and (C) combined) that lists years and the allocations for each asset to at least four significant figures. Determine if each of these portfolios is long or solvent. Comment on the implications of what you find.

(4) Assume that the safe investment for each year is the U.S. T-Bill rate available at the beginning of that year. Assume that the credit-line for each year is three points higher than the U.S. T-Bill rate. For each $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 compute the tangency portfolio allocations $\mathbf{f}_{\text{st}}$ and $\mathbf{f}_{\text{ct}}$ whenever they exist. Present these in three tables as was done in part 3. Identify when each of these portfolios exists and when it does determine if it is long or solvent. Comment on the implications of what you find.

(5) For each year graph in the $\sigma\mu$-plane the three frontiers associated with the appropriate $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 and the three efficient frontiers associated with the tangency portfolios found in part 4. Comment on the implications of what you see.

(6) Assume that the safe investment for each year is the U.S. T-Bill rate available at the beginning of that year. For each $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 compute the long tangency portfolio allocation $\mathbf{f}_{\text{lt}}$. Present these in three tables as was done in part 3.

(7) For each year graph in the $\sigma\mu$-plane the three long frontiers associated with the appropriate $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 and the three efficient long frontiers associated with the tangency portfolios found in part 6. Comment on the implications of what you see.

(8) For each year with return history $\{\mathbf{r}(d)\}_{d=1}^{D}$, compute the liquidity metrics

$$\omega_{\rm st}^{\rho} = \max\left\{0\,,\ \min\left\{\frac{1+\mathbf{r}(d)^{\rm T}\mathbf{f}_{\rm st}}{1+\mathbf{m}^{\rm T}\mathbf{f}_{\rm st}}\ :\ d=1,\cdots,D\right\}\right\}\,,$$

$$\omega_{\rm ct}^{\rho} = \max\left\{0\,,\ \min\left\{\frac{1+\mathbf{r}(d)^{\rm T}\mathbf{f}_{\rm ct}}{1+\mathbf{m}^{\rm T}\mathbf{f}_{\rm ct}}\ :\ d=1,\cdots,D\right\}\right\}\,,$$

$$\omega_{\rm lt}^{\rho} = \min\left\{\frac{1+\mathbf{r}(d)^{\rm T}\mathbf{f}_{\rm lt}}{1+\mathbf{m}^{\rm T}\mathbf{f}_{\rm lt}}\ :\ d=1,\cdots,D\right\}\,,$$

where $\mathbf{f}_{\rm st}$, $\mathbf{f}_{\rm ct}$, and $\mathbf{f}_{\rm lt}$ are for groups (A) and (B) combined and for groups (A), (B), and (C) combined. If $\mathbf{f}_{\rm st}$ or $\mathbf{f}_{\rm ct}$ does not exist or if $1+\mathbf{m}^{\rm T}\mathbf{f}_{\rm st}\le 0$ or $1+\mathbf{m}^{\rm T}\mathbf{f}_{\rm ct}\le 0$ then set the associated metric to 0. Present this data in a table.

(9) For each year compute the linkage metrics

$$\omega_k^{\lambda} = \frac{\lambda_k}{\lambda_9}\,,\qquad \text{for } k=1,\cdots,8\,,$$

where

$$\lambda_1\le\lambda_2\le\lambda_3\le\lambda_4\le\lambda_5\le\lambda_6\le\lambda_7\le\lambda_8\le\lambda_9\,,$$

are the eigenvalues of $\mathbf{V}$ for groups (A), (B), and (C) combined. Present this data in a table.

(10) By comparing each of the these metrics with VFIAX, explore if any of these metrics warn of a potential economic downturn.