Math 420, Spring 2021
Second Solo Homework:
Introduction to the Threads

Due Tuesday, 9 February, 2021

Exercise 1. Compute m_i, v_{ij}, and c_{ij} for each of the following groups of assets based on daily closing price data with uniform weights:

(i) VFIAX, VBTLX, and VGSLX in 2020;
(ii) VFIAX, VBTLX, and VGSLX in 2019 and 2020;
(iii) VFIAX, VBTLX, VGSLX, VIMAX, VTIAX, and VTABX in 2020;
(iv) VFIAX, VBTLX, VGSLX, VIMAX, VTIAX, and VTABX in 2019 and 2020.

a. Describe the assets VFIAX, VBTLX, and VGSLX. Display m_i as a 3-vector and v_{ij} and c_{ij} as 3×3-matrices for (i) and (ii). Explain the differences between these objects for groups (i) and (ii).
b. Compute a complete set of eigenpairs for the 3×3-matrices v_{ij} for groups (i) and (ii). What conclusions do you draw about these assets from the relative size of the eigenvalues?
c. Describe the assets VIMAX, VTIAX, and VTABX. Display m_i as a 6-vector and v_{ij} and c_{ij} as 6×6-matrices for (iii) and (iv). Explain the differences between these objects for groups (iii) and (iv).
d. Compute a complete set of eigenpairs for the 6×6-matrices v_{ij} for groups (iii) and (iv). What conclusions do you draw about these assets from the relative size of the eigenvalues? How do these six eigenvalues compare with the three from part (b)?
e. Give explanations for the values of c_{ij} you computed for groups (iii) and (iv).

Exercise 2 on the next page.

Exercise 2. Consider the following seven points in the 2-D plane:

$$\mathcal{V} = \{(0,1),(-1,-1),(1,0),(-1,0),(0,2),(0,-2),(1,3)\}$$

You need to write and execute a piece of (Matlab) code that performs the following:

a. Denote by R the 7×7 matrix of pairwise distances between these points. Compute and print out this matrix.

b. Let $K = 3$. For each point in this set find the closest K-neighbors and collect these edges in the set $\mathcal{E}$. Plot the geometric graph $(\mathcal{V}, \mathcal{E})$.

c. For each point in $\mathcal{V}$ find the closest point on the line $2y - x + 1 = 0$. Plot both the set $\mathcal{V}$ and their projection onto the line $2y - x + 1 = 0$.

d. For each point $(a,b) \in \mathcal{V}$ consider its projection (x,y) onto the line $2y - x + 1 = 0$ computed at part (c). Let $\mathbf{r} = (r_1, r_2) = (a,b) - (x,y)$ denote the residual vector, or the approximation error. Verify numerically that all residual vectors are parallel with each other. What direction are they parallel with?

e. Compute the scalar product (or dot product) between the unit vector parallel to the line $2y - x + 1 = 0$ and the unit vector passing through the origin and parallel to residuals computed at part (d).