Math 420, Spring 2018
Team Homework 8

due Tuesday, 4 May, 2021

In the following exercises consider the risky assets in groups (A) (B) and (C) of your final project. Use adjusted closing prices to compute the return of each asset for each trading day over the last five calendar years — namely the years ending December 31 of 2016-2020.

There are 20 quarters within this five year period. There are 17 one-year periods within these five years — the first consisting of quarters 1-4, the second consisting of quarters 2-5, and so on until the last consisting of quarters 17-20. We call the return histories over these 17 one-year periods *rolling histories* and label each by its first quarter.

For each of the 17 one-year histories compute $\mathbf{m}$ and $\mathbf{V}$ using uniform weights for the assests in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined. Use the U.S. T-Bill (13 week) rate available at the beginning of each history as the safe investment rate for the data from that period. Assume that the credit line rate for each period is three points higher than the U.S. T-Bill rate.

Exercise 1. For each of the 17 one-year histories and each of the three pairs $(\mathbf{m}, \mathbf{V})$ compute:

- the long frontier $\sigma = \sigma_{\mathrm{f}}(\mu)$ for $\mu \in [\mu_{\mathrm{mn}}, \mu_{\mathrm{mx}}]$, where
$$\mu_{\mathrm{mn}} = \min\{m_i \,:\, i = 1, \cdots, N\}\,, \quad \mu_{\mathrm{mx}} = \max\{m_i \,:\, i = 1, \cdots, N\}\,,$$
- the long tangent point $(\sigma_{\mathrm{lt}}, \mu_{\mathrm{lt}})$ on the long frontier associated with the safe investment;
- the efficient long frontier $\mu = \mu_{\mathrm{elf}}(\sigma)$ for $\sigma \in [0, \sigma_{\mathrm{mx}}]$ whenever $\mu_{\mathrm{si}} < \mu_{\mathrm{mx}}$, where
$$\sigma_{\mathrm{mx}} = \sigma_{\mathrm{f}}(\mu_{\mathrm{mx}})\,;$$
- the allocation $\mathbf{f}_{\mathrm{lt}}$ for the tangent portfolio on the long frontier associated with the safe investment.

Compute $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$, $\widehat{\mathrm{Vr}}\big(\log(1+R)\big)$, and the signal-to-noise ratio of $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$ for the portfolios with allocation $\mathbf{f}_{\mathrm{lt}}$ for each of the histories and each of the three pairs $(\mathbf{m}, \mathbf{V})$. There should be nine plots, one for each pair of the three quantities with the three portfolios. For which of these portfolios are you most certain of the expected value of its return? Give your reasoninng.

Exercise 2. For each one-year history that begins with the first or third quarter of a calendar year plot:

- the frontier hyperbola for each of the three pairs $(\mathbf{m}, \mathbf{V})$;
- the long frontier and the efficient long frontier for each of the three pairs $(\mathbf{m}, \mathbf{V})$.

There should be nine graphs, one for each one-year history that begins with the first or third quarter of a calendar year.

Exercise 3. For each of the 17 one-year histories, each of the three pairs $(\mathbf{m}, \mathbf{V})$, and each of the portfolios with allocation $\mathbf{f}_{\mathrm{lt}}$, compare $\widehat{\mathrm{Var}}\big(\log(1+R)\big)$ with the mean-variance estimators

$$\hat{\theta}_{\mathrm{q}}(\mathbf{f}) = \mathbf{f}^{\mathrm{T}}\mathbf{V}\mathbf{f}\,, \qquad \hat{\theta}_{\mathrm{t}}(\mathbf{f}) = \frac{\mathbf{f}^{\mathrm{T}}\mathbf{V}\mathbf{f}}{(1+\mathbf{m}^{\mathrm{T}}\mathbf{f})^2}\,.$$

Specifically, for each of these three portfolios, each of the one-year histories, and each of these two estimators compute the ratio

$$\frac{\sqrt{\widehat{\mathrm{Vr}}\big(\log(1+R)\big)} - \sqrt{\hat{\theta}(\mathbf{f})}}{\widehat{\mathrm{Std}}\Big(\widehat{\mathrm{Ex}}\big(\log(1+R)\big)\Big)}\,.$$

Present these ratios in three tables, one for each portfolio, where each table has two columns and 17 rows. What conclusions do you draws from these comparisions? Specifically, address how errors introduced by the various mean-variance approximations cautious objectives compare with those of the estimator $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$ that were computed in the last homework.