Math 420, Spring 2021
Team Homework 7

due Tuesday, 20 April, 2020

In the following exercises consider the risky assets in groups (A) (B) and (C) of your final project. Use adjusted closing prices to compute the return of each asset for each trading day over the last five calendar years — namely the years ending December 31 of 2016-2020.

There are 20 quarters within this five year period. There are 17 one-year periods within these five years — the first consisting of quarters 1-4, the second consisting of quarters 2-5, and so on until the last consisting of quarters 17-20. We call the return histories over these 17 one-year periods *rolling histories* and label each by its first quarter.

For each of the 17 one-year histories compute $\mathbf{m}$ and $\mathbf{V}$ using uniform weights for the assests in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined. Use the U.S. T-Bill (13 week) rate available at the beginning of each history as the safe investment rate for the data from that period. Assume that the credit line rate for each period is three points higher than the U.S. T-Bill rate.

Exercise 1. For each of the 17 one-year histories and each of the three pairs $(\mathbf{m}, \mathbf{V})$ compute

- the allocation $\mathbf{f}_{\mathrm{st}}$ for the tangent portfolio on the frontier hyperbola associated with the safe investment,
- the allocation $\mathbf{f}_{\mathrm{ct}}$ for the tangent portfolio on the frontier hyperbola associated with the credit line,
- the allocation $\mathbf{f}_{\mathrm{lt}}$ for the tangent portfolio on the long frontier associated with the safe investment.

Compute the signal-to-noise ratio of $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$ for the portfolios with allocations $\mathbf{f}_{\mathrm{st}}$, $\mathbf{f}_{\mathrm{ct}}$, and $\mathbf{f}_{\mathrm{lt}}$ for each of the histories and each of the three pairs $(\mathbf{m}, \mathbf{V})$. For each portfolio plot this ratio as a function of the first quarter of each history. There should be nine plots, one for each portfolio. How do these ratios compare with those of the individual assets? For which of these portfolios are you most certain of the expected value of its return? Give your reasoninng.

Exercise 2. For each of the 17 one-year histories, each of the three pairs $(\mathbf{m}, \mathbf{V})$, and each of the portfolios with allocations $\mathbf{f}_{\mathrm{st}}$, $\mathbf{f}_{\mathrm{ct}}$, and $\mathbf{f}_{\mathrm{lt}}$, compare $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$ with the mean-variance estimators $\hat{\gamma}_{\mathrm{p}}(\mathbf{f})$, $\hat{\gamma}_{\mathrm{q}}(\mathbf{f})$, $\hat{\gamma}_{\mathrm{r}}(\mathbf{f})$, $\hat{\gamma}_{\mathrm{s}}(\mathbf{f})$, $\hat{\gamma}_{\mathrm{t}}(\mathbf{f})$. Specifically, each of the nine portfolios, each of the one-year histories, and each of these five estimators compute the ratio

$$\frac{\widehat{\mathrm{Ex}}\big(\log(1+R)\big) - \hat{\gamma}(\mathbf{f})}{\widehat{\mathrm{Std}}\Big(\widehat{\mathrm{Ex}}\big(\log(1+R)\big)\Big)},$$

the ratio of the mean-variance approximation error to the growth-rate noise. Present these ratios in nine tables, one for each portfolio, where each table has five columns and 17 rows. How do these ratios compare with the signal-to-noise ratios computed in Exercise 1? What conclusions do you draws from these comparisions? Specifically, address how errors introduced by the various mean-variance approximations compare with those of the estimator $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$, and which of the mean-variance estimators seem most reasonable.