

Math 420, Spring 2021

Team Homework 6

due Tuesday, 13 April, 2021

In the following exercises consider the risky assets in groups (A) (B) and (C) of your final project. Consider one-year histories of daily share price data for each asset over the years ending December 31 of 2016-2020. Use adjusted closing prices to compute the return for each trading day over these five years.

There are 20 quarters within this five year period. There are 17 one-year periods within these five years — the first consisting of quarters 1-4, the second consisting of quarters 2-5, and so on until the last consisting of quarters 17-20. We call the return histories over these 17 one-year periods *rolling histories* and label each by its first quarter.

For each of the 17 one-year histories compute \mathbf{m} and \mathbf{V} for the assets in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined using uniform weights. Use the U.S. T-Bill (13 week) rate available at the beginning of each history as the safe investment for the data from that period. Assume that the credit line for each period is three points higher than the U.S. T-Bill rate.

Exercise 1. For each of the five calendar years plot a graph that shows

- the points (m_i, σ_i) associated with each asset,
- the frontier hyperbola associated with each of the three pairs (\mathbf{m}, \mathbf{V}) ,
- the points (μ_{st}, σ_{st}) and (μ_{ct}, σ_{ct}) on the frontier hyperbola associated with each of the three pairs (\mathbf{m}, \mathbf{V}) ,
- the efficient frontier associated with each of the three pairs (\mathbf{m}, \mathbf{V}) ,
- the long frontier associated with each of the three pairs (\mathbf{m}, \mathbf{V}) ,
- the point (μ_{lt}, σ_{lt}) on the long frontier associated with each of the three pairs (\mathbf{m}, \mathbf{V}) ,
- the efficient long frontier associated with each of the three pairs (\mathbf{m}, \mathbf{V}) .

Exercise 2. For each of the 17 one-year histories and each of the three pairs (\mathbf{m}, \mathbf{V}) compute

- the allocation \mathbf{f}_{st} for the tangent portfolio on the frontier hyperbola associated with the safe investment,
- the allocation \mathbf{f}_{ct} for the tangent portfolio on the frontier hyperbola associated with the credit line,
- the allocation \mathbf{f}_{lt} for the tangent portfolio on the long frontier associated with the safe investment.

Compute the signal-to-noise ratio of $\hat{\mu}$ for the portfolios with allocations \mathbf{f}_{st} , \mathbf{f}_{ct} , and \mathbf{f}_{lt} for each of the histories and each of the three pairs (\mathbf{m}, \mathbf{V}) . For each portfolio plot this ratio as a function of the first quarter of each history. There should be nine plots, one for each portfolio. How do these ratios compare with those of the individual assets? For which of these portfolios are you most certain of the expected value of its return? Give your reasoning.

Exercise 3. Compute the metrics ω^m , ω^v , ω^{KS} , ω^{ar} , and ω^{ac} to the portfolios with allocations \mathbf{f}_{st} , \mathbf{f}_{ct} , and \mathbf{f}_{lt} for each of the histories and each of the three pairs (\mathbf{m}, \mathbf{V}) . For ω^m , ω^v , and ω^{KS} compare each quarter with every other quarter in its history, so there are six comparisons for each history. For each portfolio plot these five metrics as a function of the first quarter of each history. There should be nine plots, one for each portfolio. How do these metrics compare with those of the individual assets? Which of these portfolios is best described by an IID model? Give your reasoning.