Math 420, Spring 2021
Team Homework 4

due Tuesday, 30 March, 2021

Recall that if $\{R_d\}_{d=1}^D$ are values drawn from an IID process then given any function $\psi : (-1, \infty) \to \mathbb{R}$ and any positive weights $\{w_d\}_{d=1}^D$ that sum to 1, unbiased estmators for the expected value and variance of $\Psi = \psi(R)$ are given by

$$\widehat{\mathrm{Ex}}(\Psi) = \sum_{d=1}^{D} w_d \, \Psi_d \,, \qquad \widehat{\mathrm{Var}}(\Psi) = \frac{1}{1 - \bar{w}} \sum_{d=1}^{D} w_d \left(\Psi_d - \widehat{\mathrm{Ex}}(\Psi) \right)^2 \,,$$

where $\Psi_d = \psi(R_d)$ and

$$\bar{w} = \sum_{d=1}^{D} w_d^2 \,.$$

Then a (biased) estimator of the standard deviation of $\widehat{\mathrm{Ex}}(\Psi)$ is

$$\widehat{\mathrm{Std}}\Big(\widehat{\mathrm{Ex}}(\Psi)\Big) = \sqrt{\bar{w}} \sqrt{\widehat{\mathrm{Var}}(\Psi)} \,.$$

In the following exercises consider the risky assets in groups (A), (B), and (C) of your Project Two. For each asset consider its daily return history $\{r(d)\}_{d=1}^D$ over each of the five years over the period 2016-2020. Treat these values as if they are drawn from an IID process.

Exercise 1. For each asset and each year use uniform weights to compute and compare $\widehat{\mathrm{Ex}}(R)$ and $\widehat{\mathrm{Std}}\Big(\widehat{\mathrm{Ex}}(R)\Big)$. The signal-to-noise ratio for $\widehat{\mathrm{Ex}}(R)$ is

$$\frac{\widehat{\mathrm{Ex}}(R)}{\widehat{\mathrm{Std}}\Big(\widehat{\mathrm{Ex}}(R)\Big)} \,.$$

Based on this, for each year order the nine assets from that with the greatest certainity to least certainty in $\widehat{\mathrm{Ex}}(R)$. Can you explain the order?

Exercise 2. For each asset and each year use uniform weights to compute and compare $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$ and $\widehat{\mathrm{Std}}\Big(\widehat{\mathrm{Ex}}\big(\log(1+R)\big)\Big)$. The signal-to-noise ratio for $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$ is

$$\frac{\widehat{\mathrm{Ex}}\big(\log(1+R)\big)}{\widehat{\mathrm{Std}}\Big(\widehat{\mathrm{Ex}}\big(\log(1+R)\big)\Big)} \,.$$

Based on this, for each year order the nine assets from that with the greatest certainity to least certainty in $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$. (Here log is the natural logorithm.) Compare this ordering with the one that you found in Problem 1.

Exercise 3. What relationship do you observe between $\widehat{\mathrm{Ex}}(R)$ and $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$? Can you explain it? (Hint: Jensen Inequality.)