

Math 420, Spring 2021

Team Homework 3

Limited-Leverage Frontiers

due Thursday, 4 March, 2021

Exercise 1. Use adjusted closing prices to compute the return for each trading day over the three years 2018-2020. Compute \mathbf{m} and \mathbf{V} for the assets in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined using one-year histories with uniform weights and daily data for each the three years ending 31 December 2018-2020. Print out the three \mathbf{m} and \mathbf{V} for each of the three years 2018-2020 to four significant digits. What relationship do the three \mathbf{m} and \mathbf{V} for each year have to each other? (This is the same exercise as the last two times!)

Exercise 2. The U.S. T-Bill rate available at the end of each year as the safe investment for the data from that year. Assume that the credit-line for each year is three points higher than the U.S. T-Bill rate. For each \mathbf{m} and \mathbf{V} computed in Exercise 1 compute the tangency portfolio allocations \mathbf{f}_{st} and \mathbf{f}_{ct} whenever they exist. Present these in six tables (two for group (A), two for groups (A) and (B) combined, and two for groups (A), (B), and (C) combined) that lists years, the allocations for each asset rounded to the nearest thousandth, and its leverage. For each tangent portfolio that exists determine whether or not it is long and whether or not it is solvent. Comment on the implications of what you find.

Exercise 3. For each year and each of the three collections of assets (A, AB, ABC) graph in the $\sigma\mu$ -plane:

- the volatility and return mean for that year of each risky and risk-free asset;
- the volatility and return mean for that year of the Markowitz portfolio with allocation that is equidistributed within the collection of risky assets;
- the volatility and return mean for that year of the minimum volatility portfolio;
- the efficient frontier that is associated with the appropriate \mathbf{m} and \mathbf{V} were computed in Exercise 1 and with the risk-free assets from Exercise 2;
- the efficient long frontier that is associated with the appropriate \mathbf{m} and \mathbf{V} were computed in Exercise 1 and with the safe investment from Exercise 2;
- the limited frontier for leverage limit $\ell = 1$ that is associated with the appropriate \mathbf{m} and \mathbf{V} were computed in Exercise 1;
- the limited frontier for leverage limit $\ell = 5$ that is associated with the appropriate \mathbf{m} and \mathbf{V} were computed in Exercise 1.

There should be 9 graphs (one for each year and each collection of assets) with either three, six, or nine assets, two portfolios, one efficient frontier, one efficient long frontier, two limited frontiers, and all the tangent portfolios of the two efficient frontiers plotted. Use different symbols or colors to distinguish points associated with the different groups (A), (B), and (C). Comment on any relationships that you see between the objects plotted on each graph. (This will be easier to do if you use the same scales for each of the graphs. Each σ -axis should begin at $\sigma = 0$.)