Math 420, Spring 2021
Team Homework 2
Efficient and Long Frontiers

due Tuesday, 23 February, 2021

Exercise 1. Use adjusted closing prices to computer the return for each trading day over the three years 2018-2020. Compute $\mathbf{m}$ and $\mathbf{V}$ for the assets in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined using one-year histories with uniform weights and daily data for each the three years ending 31 December 2018-2020. Print out the three $\mathbf{m}$ and $\mathbf{V}$ for each of the three years 2018-2020 to four significant digits. What relationship do the three $\mathbf{m}$ and $\mathbf{V}$ for each year have to each other?

Exercise 2. For each $\mathbf{m}$ and $\mathbf{V}$ computed in Exercise 1, compute the tangent portfolio allocations $\mathbf{f}_{\text{st}}$ and $\mathbf{f}_{\text{ct}}$ associated with the risk-free rates μ_{si} and μ_{cl} computed from the U.S. Treasury Bill rate on the last trading day of each of the years 2018-2020 as discussed in the slides. Recall that these will exist only if the risk-free rate is less than minimum volatility rate for the associated $\mathbf{m}$ and $\mathbf{V}$. Present these in six tables (two for group (A), two for groups (A) and (B) combined, and two for groups (A), (B), and (C) combined) that lists years and the allocations for each asset rounded to the nearest thousandth. For each tangent portfolio that exists determine whether or not it is long and whether or not it is solvent. Comment on how these change from year to year for the same groupings of assets. For example: Do these tangent portfolios exist? Do they hold short positions? Are they solvent? Can you explain what you see from what you know about the individual assets?

Exercise 3. For each year graph in the $\sigma\mu$-plane:

- the three frontiers associated with the appropriate $\mathbf{m}$ and $\mathbf{V}$ from Exercise 1;
- the three long frontiers associated with the appropriate $\mathbf{m}$ and $\mathbf{V}$ from Exercise 1;
- the three efficient frontiers associated with the appropriate $\mathbf{m}$ and $\mathbf{V}$ from Exercise 1 and the appropriate tangent portfolios from Exercise 2;
- the volatility and return mean of each appropriate tangent portfolio from Exercise 2;
- the volatility and return mean for that year of each asset;

There should be 3 graphs — one for each year — each with three frontiers, three long frontiers, three efficient frontiers, nine assets, and up to six tangent portfolios plotted. Use different symbols or colors to distinguish points associated with the different groups (A), (AB), and (ABC). Comment on any relationships that you see between the objects plotted on each graph. (This will be easier to do if you use the same scales for each of the graphs. Each σ-axis should begin at $\sigma = 0$.) What relationships do you see between the various frontiers, long frontiers, efficient frontiers, and assets? Each of these graphs should fill most of a page.