

Math 420, Spring 2021 Fifth Team Homework

Consider the datafile assigned to this homework.

Your task is to study the two random graph models: Erdős-Renyi and SSBM. In each case you need to estimate the model parameters (p and a, b , respectively) and compare the estimated number of q -cliques given the model to the actual number of q -cliques (with $q = 3$ and $q = 4$).

1. Write a Matlab function that computes the number of 4-cliques of a given graph. Follow the same preamble as in the previous homework.

2. Use your Matlab function to compute the following statistics: m , the number of edges; t , the number of 3-cliques (triangles); and f , the number of 4-cliques (complete four-vertex subgraphs).

3. For the Erdős-Renyi model $\mathcal{G}_{n,p}$ determine the Maximum Likelihood Estimate for p that best fits the dataset.

4. Compute the Expectation of the numbers of 3-cliques $X_{3,ER} = \mathcal{E}[X_3; p]$, and of 4-cliques $X_{4,ER} = \mathcal{E}[X_4; p]$ under the Erdős-Renyi model constructed at part 3.

5. For same dataset, compute the estimated parameters a, b for the Symmetric Stochastic Block Model with 2-communities $SSBM(n, 2, a, b)$ using the Method of Moments.

6. Compute the Expectation of the number of 4-cliques $X_{4,SSBM} = \mathcal{E}[X_4; a, b]$ under the $SSBM(n, 2, a, b)$ model constructed at part 5. You may need to do some analytic computations (i.e., simplify expressions) in addition to what we do in class.