Math 420, Spring 2020
Project Two: Validity of the IID Model

presentation due Tuesday, 12 May, 2020

report due Friday, 15 May, 2020

This project explores how metrics that test the valitity of the IID model might guide the choice of the caution coefficient χ.

Use daily adjusted closing prices for each of the risky asset in your assigned groups (A) (B) and (C) to compute the return for each trading day over the fifteen years 2005-2019. For each of the fifteen years ending December 31 of 2005-2019 compute $\mathbf{m}$ and $\mathbf{V}$ for the assests in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined using one-year histories with uniform weights. Use the U.S. T-Bill rate available at the beginning of each year as the safe investment for the data from that year. Assume that the credit line for each year is three points higher than the U.S. T-Bill rate.

For each of the fifteen years and each of the three pairs $(\mathbf{m}, \mathbf{V})$ compute

- $\mathbf{f}_{\mathrm{si}}$, the allocation for the tangent portfolio on the frontier hyperbola associated with the safe investment,
- $\mathbf{f}_{\mathrm{cl}}$, the allocation for the tangent portfolio on the frontier hyperbola associated with the credit line,
- $\mathbf{f}_{\mathrm{ls}}$, the allocation for the tangent portfolio on the long frontier associated with the safe investment.

Apply the metrics ω^{m}, ω^{v}, ω^{KS}, ω^{ar}, and ω^{ac} to the portfolios with allocations $\mathbf{f}_{\mathrm{si}}$ $\mathbf{f}_{\mathrm{cl}}$, and $\mathbf{f}_{\mathrm{ls}}$ for each of the fifiteen years and each of the three pairs $(\mathbf{m}, \mathbf{V})$. For each portfolios plot these five metrics as a function of quarters. (There are 60 quarters over the fifteen year history.) There should be nine plots, one for each portfolio, each with five metrics plotted. Identify one or two metrics that you think are most informative about the validity of the IID model.

The reasonable estimator of the cautious objective is

$$\widehat{\Gamma}_{\mathrm{r}}^{\chi}(\mathbf{f}) = \log\bigl(1 + \hat{\mu}(\mathbf{f})\bigr) - \tfrac{1}{2}\mathbf{f}^{\mathrm{T}}\mathbf{V}\mathbf{f} - \chi\sqrt{\mathbf{f}^{\mathrm{T}}\mathbf{V}\mathbf{f}}\,.$$

Let the caution coefficient χ take the form

$$\chi = \zeta\sqrt{\frac{\bar{w}}{1-\bar{w}}}\,.$$

For each of the fourteen years 2005-2018 find the long portfolio allocation of the six assets in group A and B combined that maximizes this objective for when $\zeta = 0$, .25, .5, .75, 1, 1.25, 1.5, 1.75 and 2. (This is a total of $14 \times 9 = 126$ optimization problems. For each of these fourteen years determine which of these nine values of ζ yields the allocation with best performance in the *subsequent* year — i.e. determine which of the nine optimal allocations $\mathbf{f}$ yields the maximum value of

$$\sum_{d=1}^{D} \log\bigl(1 + r(d, \mathbf{f})\bigr)\,,$$

where the sum is over the data from the year subsequent to the one used to compute the nine optimal allocations $\mathbf{f}$. Use scatter plots to seek correlations between these best ζ and the metrics that you chose above.