Math 420, Spring 2020
Second Solo Homework:
Introduction to the Threads

Due Tuesday, 11 February, 2020

Exercise 1. Compute m_i, v_{ij}, and c_{ij} for each of the following groups of assets based on daily closing price data with uniform weights:

(i) VFINX, VBMFX, and VGSIX in 2019;
(ii) VFINX, VBMFX, and VGSIX in 2018 and 2019;
(iii) VFINX, VBMFX, VGSIX, VGTSX, VMVIX, and VTIBX in 2019;
(iv) VFINX, VBMFX, VGSIX, VGTSX, VMVIX, and VTIBX in 2018 and 2019.

a. Describe the assets VFINX, VBMFX, and VGSIX. Display m_i as a 3-vector and v_{ij} and c_{ij} as 3×3-matrices for (i) and (ii). Explain the differences between these objects for groups (i) and (ii).
b. Compute a complete set of eigenpairs for the 3×3-matrices v_{ij} for groups (i) and (ii). What conclusions do you draw from the eigenvalues about the dimensionality of these assets?
c. Describe the assets VGTSX, VMVIX, and VTIBX. Display m_i as a 6-vector and v_{ij} and c_{ij} as 6×6-matrices for (iii) and (iv). Explain the differences between these objects for groups (iii) and (iv).
d. Compute a complete set of eigenpairs for the 6×6-matrices v_{ij} for groups (iii) and (iv). What conclusions do you draw from the eigenvalues about the dimensionality of these assets?
e. Give explanations for the values of c_{ij} you computed for groups (iii) and (iv).

Exercise 2 on the next page.

Exercise 2.

Consider the following seven points in the 2-D plane:

$$\mathcal{V} = \{(1,0),(-1,-1),(0,1),(0,-1),(2,0),(-2,0),(3,1)\}$$

You need to write and execute a piece of (Matlab) code that performs the following:

a. Denote by R the 7×7 matrix of pairwise distances between these points. Compute and print out this matrix.

b. Let $K = 3$. For each point in this set find the closest K-neighbors and collect these edges in the set $\mathcal{E}$. Plot the geometric graph $(\mathcal{V}, \mathcal{E})$.

c. For each point in $\mathcal{V}$ find the closest point on the line $y = 2x + 1$. Plot both the set $\mathcal{V}$ and their projection onto the line $y = 2x + 1$.

d. For each point $(a, b) \in \mathcal{V}$ consider its projection (x, y) onto the line $y = 2x + 1$ computed at part (c). Let $\mathbf{r} = (r_1, r_2) = (a, b) - (x, y)$ denote the residual vector, or the approximation error. Verify numerically that all residual vectors are parallel with each other. What direction are they parallel with?

e. Compute the scalar product (or dot product) between the unit vector parallel to the line $y = 2x + 1$ and the unit vector parallel to residuals computed at part (d).