Math 420, Spring 2020
Seventh Team Homework

due Thursday, 30 April, 2020

In the following exercises consider the risky assets in groups (A) (B) and (C) of your final project. Consider one-year histories of daily share price data for each asset over the years ending December 31 of 2015-2019. There are 20 quarters within this five year period.

Use adjusted closing prices to compute the return for each trading day over the five years 2015-2019. For each of the five years ending December 31 of 2015-2019 compute $\mathbf{m}$ and $\mathbf{V}$ for the assests in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined using one-year histories with uniform weights. Use the U.S. T-Bill rate available at the beginning of each year as the safe investment for the data from that year. Assume that the credit line for each year is three points higher than the U.S. T-Bill rate.

For each of the five years and each of the three pairs $(\mathbf{m}, \mathbf{V})$ compute

- $\mathbf{f}_{\mathrm{si}}$, the allocation for the tangent portfolio on the frontier hyperbola associated with the safe investment,
- $\mathbf{f}_{\mathrm{cl}}$, the allocation for the tangent portfolio on the frontier hyperbola associated with the credit line,
- $\mathbf{f}_{\mathrm{ls}}$, the allocation for the tangent portfolio on the long frontier associated with the safe investment.

Exercise 1. For each of the nine portfolios and each year use uniform weights to compute and compare $\widehat{\mathrm{Ex}}(R)$ and $\widehat{\mathrm{Std}}\Big(\widehat{\mathrm{Ex}}(R)\Big)$. The signal-to-noise ratio for $\widehat{\mathrm{Ex}}(R)$ is

$$\frac{\widehat{\mathrm{Ex}}(R)}{\widehat{\mathrm{Std}}\Big(\widehat{\mathrm{Ex}}(R)\Big)}\,.$$

Using this, for each year identify the portfolio with the greatest certainity in $\widehat{\mathrm{Ex}}(R)$.

Exercise 2. For each of these nine portfolios and each year use uniform weights to compute and compare $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$ and $\widehat{\mathrm{Std}}\Big(\widehat{\mathrm{Ex}}\big(\log(1+R)\big)\Big)$. The signal-to-noise ratio for $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$ is

$$\frac{\widehat{\mathrm{Ex}}\big(\log(1+R)\big)}{\widehat{\mathrm{Std}}\Big(\widehat{\mathrm{Ex}}\big(\log(1+R)\big)\Big)}\,.$$

Using this, for each year identify the portfolio with the greatest certainity in $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$. (Here log is the natural logorithm.)

Exercise 3. For each of these nine portfolios and each year to compare $\widehat{\mathrm{Ex}}\big(\log(1+R)\big)$ with the mean-variance estimators $\hat{\gamma}_{\mathrm{p}}(\mathbf{f})$, $\hat{\gamma}_{\mathrm{q}}(\mathbf{f})$, $\hat{\gamma}_{\mathrm{r}}(\mathbf{f})$, $\hat{\gamma}_{\mathrm{s}}(\mathbf{f})$, $\hat{\gamma}_{\mathrm{t}}(\mathbf{f})$. Specifically, for each of these nine portfolios, each year, and each of these five estimators compute the ratio

$$\frac{\widehat{\mathrm{Ex}}\big(\log(1+R)\big) - \hat{\gamma}(\mathbf{f})}{\widehat{\mathrm{Std}}\Big(\widehat{\mathrm{Ex}}\big(\log(1+R)\big)\Big)}\,,$$

the ratio of the mean-variance approximation error to the callibration uncertainty.