

Math 420, Spring 2020

Third Team Homework

Limited-Leverage Efficient Frontiers

due Tuesday, 3 March, 2020

Exercise 1. Use adjusted closing prices to compute the return for each trading day over the three years 2017-2019. Compute \mathbf{m} and \mathbf{V} for the assets in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined using one-year histories with uniform weights and daily data for each the three years ending 31 December 2017-2019. Print out the three \mathbf{m} and \mathbf{V} for each of the three years 2017-2019 to four significant digits. What relationship do the three \mathbf{m} and \mathbf{V} for each year have to each other? (This is the same exercise as the last two times!)

Exercise 2. The U.S. T-Bill rate available at the end of each year as the safe investment for the data from that year. Assume that the credit-line for each year is three points higher than the U.S. T-Bill rate. For each \mathbf{m} and \mathbf{V} computed in Exercise 1 compute the tangency portfolio allocations \mathbf{f}_{st} and \mathbf{f}_{ct} whenever they exist. Present these in six tables (two for group (A), two for groups (A) and (B) combined, and two for groups (A), (B), and (C) combined) that lists years, the allocations for each asset rounded to the nearest thousandth, and its leverage. Comment on the implications of what you find.

Exercise 3. For each year and each of the three collections of assets (A, AB, ABC) graph in the $\sigma\mu$ -plane:

- the volatility and return mean for that year of each risky and risk-free asset;
- the volatility and return mean for that year of the Markowitz portfolio with allocation that is equidistributed within the collection of risky assets;
- the volatility and return mean for that year of the minimum volatility portfolio;
- the efficient frontier that is associated with the appropriate \mathbf{m} and \mathbf{V} were computed in Exercise 1 and with the risk-free assets from Exercise 2;
- the long efficient frontier that is associated with the appropriate \mathbf{m} and \mathbf{V} were computed in Exercise 1 and with the safe investment from Exercise 2;
- the limited efficient frontier for leverage limit $\ell = 1$ that is associated with the appropriate \mathbf{m} and \mathbf{V} were computed in Exercise 1 and with the risk-free assets from Exercise 2;
- the limited efficient frontier for leverage limit $\ell = 5$ that is associated with the appropriate \mathbf{m} and \mathbf{V} were computed in Exercise 1 and with the risk-free assets from Exercise 2.

There should be 9 graphs (one for each year and each collection of assets) with either three, six, or nine assets, two portfolios, four efficient frontiers, and all the tangent portfolios of the four efficient frontiers plotted. Use different symbols or colors to distinguish points associated with the different groups (A), (B), and (C). Comment on any relationships that you see between the objects plotted on each graph. (This will be easier to do if you use the same scales for each of the graphs. Each σ -axis should begin at $\sigma = 0$.)