Math 420, Spring 2020
Sixth Team Homework

Exercise 1. [4pts] Write a function that computes the second smallest eigenvalue λ_1 of the normalized graph Laplacian for a given graph with n vertices. Then write a script that uses the same dateset you used last time, and computes the sequence of second smallest eigenvalue $\lambda_1(k)$ of the cumulative graph $Edges(1:k, 1:2)$, where $1 \leq k \leq m$ denotes the running number of edges. Specifically, order the edges according to the weight, starting with the largest weight first and then continue in a monotonic decreasing order. You may find easier to firstly create a data file, say graph.dat, from the raw data file assigned to your homework, that lists the edges in the appropriate order, and has the following format:

```
First line: n m
Second line: Edge1Vertex1 Edge1Vertex2
Third line: Edge2Vertex1 Edge2Vertex2
...
m+1st line: EdgemVertex1 EdgemVertex2
```

Exercise 2. [6pts] Consider the weighted undirected graph inserted below.

1. [2pts] Write down the weight matrix W, the weighted graph Laplacian $\Delta = D - W$, and the normalized weighted graph Laplacian $\tilde{\Delta}$. Compute its eigenvalues and eigenvectors.

2. [2pts] Write a function that computes the Cheeger constant and the optimal partition for a given weight matrix W, and apply it to this graph. Determine both the optimal partition and the Cheeger constant h_G.

3. [2pts] Use the second smallest eigenpair obtained at the first part to determine an alternate partition (what we called in class the "initialization"). Find the value of the criterion minimized by the Cheeger constant and compare it to h_G.

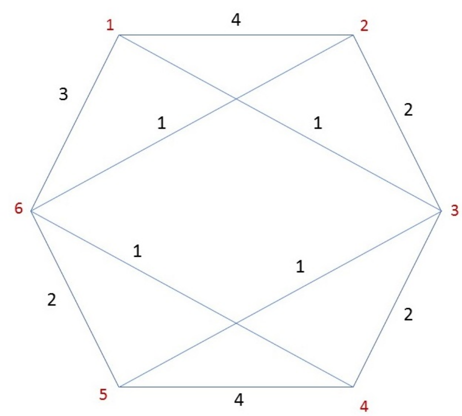

Figure 1: A weighted graph with $n = 6$ vertices. The vertex labels are marked in red. The edge weights are in black.