Math 420, Spring 2020
First Team Homework

due Tuesday, 18 February 2020

I. (8pts) Consider the text files Pair_psb420.dist and MeasuredPair_psb420.dist, both attached to this homework. The files have the following format:

```
line 1: n
line 2: d11
line 3: d12
line 4: d13
...
line n^2+1: dnn
```

where n denotes the number of vertices of a geometric graph, $d11$,...,dnn represents the pairwise distances between the n points. Note the following: the file Pair_psb420.dist contains the noiseless distances (in particular, $dii = 0$); the file MeasuredPair_psb420.dist contains noisy measurements of these distances (hence no guarantee of symmetry or positivity).

Write a Matlab script that performs the following tasks, and apply separately on these two files

1. Read-in the file and create the matrix R of pairwise distances and S of squared-pairwise distances ($S_{k,j} = R_{k,j}^2$);
2. Apply Algorithm 1 to compute the estimated Gramm matrix G;
3. Plot the eigenvalues of G; Print out the first 5 largest eigenvalues;
4. Apply Algorithm 2 to determine a 3-dimensional embedding of this geometric graph; call Y the $3 \times n$ matrix of coordinates; plot3D the point cloud and print out the figure;
5. Compute the pairwise distances between the 3-D points contained in Y: Let $\hat{R}$ be the $n \times n$ matrix whose (k, j) entry is

   $$\hat{R}_{k,j} = \|Y(1:n,k) - Y(1:n,j)\|$$

   Detemine and print the norm $\|R - \hat{R}\|_F$;
6. Compute $\varepsilon = \|G - Y^T Y\|_F$, the approcimation error; print the result on screen;
7. Compute $\sigma = \sqrt{\sum_{k=4}^{n} \lambda_k^2}$ and print out the result; here, $\lambda_4 \geq \lambda_5 \geq \cdots \geq \lambda_n$ are the smallest $n-3$ eigenvalues of G;
8. Compare ε with σ.

2. (2pts) Denote by Y_{clean} and Y_{noisy} the two estimates matrices of ccordinates obtained by your code at part 1 when run respectively on Pair_psb420.dist and MeasuredPair_psb420.dist. Compute the Frobenious norm $\|Y_{clean} - Y_{noisy}\|_F$.