Math 420, Spring 2019
First Project: Leveraged Portfolios

presentation Tuesday, 12 March, 2019
report due Thursday, 14 March, 2019

This project explores frontiers for leveraged portfolios. Consider the following groups of assets.

(A) VFINX, VBMFX, VGSLX.
(B) from the "Projects" webpage.
(C) from the "Projects" webpage.

(1) Identify the funds in (A) and (B) and describe their holdings. (This information should inform some of your subsequent answers.)

(2) For each of the years ending December 31 of 2004-2018 use one-year histories with uniform weights to compute $\mathbf{m}$ and $\mathbf{V}$ for the risky assets in group (A), in groups (A) and (B) combined, and in groups (A), (B), and (C) combined.

(3) For each $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 compute the minimum volatility portfolio allocation $\mathbf{f}_{\text{mv}}$. Present these in three tables (one for group (A), one for groups (A) and (B) combined, and one for groups (A), (B), and (C) combined) that lists years and the allocations for each asset rounded to the nearest thousandth. Determine if each of these portfolios is long or solvent. Comment on the implications of what you find.

(4) Assume that the safe investment for each year is the U.S. T-Bill rate available at the beginning of that year. Assume that the credit-line for each year is three points higher than the U.S. T-Bill rate. For each $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 compute the tangency portfolio allocations $\mathbf{f}_{\text{st}}$ and $\mathbf{f}_{\text{ct}}$ whenever they exist. Present these in three tables as was done in part 3. Identify when each of these portfolios exists and when it does determine if it is long or solvent. Comment on the implications of what you find.

(5) For each year graph in the $\sigma\mu$-plane the three frontiers associated with the appropriate $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 and the three efficient frontiers associated with the tangency portfolios found in part 4. Comment on the implications of what you see.

(6) In a similar manner, for each year graph the efficient long frontiers, for the risky assets in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined, with a safe investment of U.S. T-Bills. Comment on the implications of what you see.

(7) In a similar manner, for each year graph the efficient frontiers for leveraged portfolios with leverage $\ell = 1$ and $\ell = 5$ for the risky assets in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined, both with and without risk-free assets. For each year determine if these leveraged portfolios are all solvent over the history that you are using to calibrate that year. Comment on the implications of what you find.

(8) Present one table that for each year gives the metrics

$$\omega^{\nu}_{st} = \frac{\nu_{\rm as}}{\nu_{\rm st}}\,, \qquad \omega^{\nu}_{ct} = \frac{\nu_{\rm as}}{\nu_{\rm ct}}\,,$$

for each of the three efficient frontiers graphed in part 5. Comment on the implications of what you find.

(9) Let $\ell(\rho)$ be defined over a given history by

$$\ell(\rho) = \min\left\{ \frac{\rho_{{\rm mn}(d)} - \rho}{\rho_{\rm mx}(d) - \rho_{{\rm mn}(d)}} \,:\, d = 1, \cdot, D \right\}\,,$$

where

$$\rho_{{\rm mn}(d)} = \min\{\rho_i(d) \,:\, i = 1, \cdot, N\}\,, \qquad \rho_{{\rm mx}(d)} = \max\{\rho_i(d) \,:\, i = 1, \cdot, N\}\,.$$

Present one table that for each quarter gives the metrics

$$\omega^{\ell}_{\rho} = \frac{\ell(\rho)}{1 + \ell(\rho)}\,,$$

for $\rho = 0$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{8}$. Comment on the implications of what you find.