Math 420, Spring 2019
Third Team Homework

due Tuesday, 5 March, 2019

For the datasets assigned to this homework, 'AlignmentData_XFile.txt' and 'AlignmentData_YFile.txt', create a dynamic alignment of the two point clouds, X onto Y, using two interpolation algorithms:

1. First compute the optimal orthogonal matrix $\hat{Q}$, translation vector $\hat{z}$ and scaling factor $\hat{a}$, as in previous homework.

2. Compute and store the approximation error:

$$Er = Y - \hat{a}\hat{Q}(X - \hat{z}1^T)$$

3. Loop for $t = 0 : step : 1$ with a stepsize $step = 0.001$:

3.1 Interpolate a and z according to:

$$a(t) = (1 - t) + t\hat{a} \quad , \quad z(t) = t\hat{z};$$

3.2 Compute $Q(t)$ according to either algorithm 1 or algorithm 2 as studies in class; 3.3 Apply the translation-rotation-dilation on X point cloud:

$$X(t) = a(t)Q(t)(X - z(t)1^T)$$

3.4 Interpolate the approximation error Er and create the $Y(t)$ point cloud:

$$Y(t) = tErr + X(t).$$

4. Save $(X(t))_t$ and $(Y(t))_t$ as two movie files.

This homework should produce four (4) movies: two movieX and movieY, one for each algorithm:

Algorithm 1:

Determine an appropriate antisymmetric matrix M and orthogonal matrix J so that

$$Q(t) = Jexp(tM)$$

Algorithm 2:

In the Full alignment algorithm, compute:

$$R(t) = (1 - t)I_d + t\tilde{X}\tilde{Y}^T$$

Then use the SVD algorithm to produce $U(t)$ and $V(t)$ from $R(t) = U(t)\Sigma(t)V(t)^T$ and set

$$Q(t) = V(t)U(t)^T.$$