Math 420, Spring 2018
First Project: Solvent Portfolios

presentation Tuesday, 13 March, 2018
report due Thursday, 15 March, 2018

This project explores when frontier portfolios are solvent. Consider the following groups of assets.

(A) VFINX, VBTIX, VGSLX.
(B) from the "Projects" webpage.
(C) from the "Projects" webpage.

(1) Identify the funds in (A) and (B) and describe their holdings. (This information should inform some of your subsequent answers.)

(2) For each of the years ending December 31 of 2002-2016 use one-year histories with uniform weights to compute $\mathbf{m}$ and $\mathbf{V}$ for the risky assets in group (A), in groups (A) and (B) combined, and in groups (A), (B), and (C) combined.

(3) For each $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 compute the minimum volatility portfolio allocation $\mathbf{f}_{\text{mv}}$. Present these in three tables (one for group (A), one for groups (A) and (B) combined, and one for groups (A), (B), and (C) combined) that lists years and the allocations for each asset rounded to the nearest thousandth. Determine if each of these portfolios is long or solvent. Comment on the implications of what you find.

(4) Assume that the safe investment for each year is the U.S. T-Bill rate available at the beginning of that year. Assume that the credit-line for each year is three points higher than the U.S. T-Bill rate. For each $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 compute the tangency portfolio allocations $\mathbf{f}_{\text{st}}$ and $\mathbf{f}_{\text{ct}}$ whenever they exist. Present these in three tables as was done in part 3. Identify when each of these portfolios exists and when it does determine if it is long or solvent. Comment on the implications of what you find.

(5) For each year graph in the $\sigma\mu$-plane the three frontiers associated with the appropriate $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 and the three efficient frontiers associated with the tangency portfolios found in part 4. Comment on the implications of what you see.

(6) In a similar manner, for each year graph the efficient long frontiers, for the risky assets in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined, with a safe investment of U.S. T-Bills. Comment on the implications of what you see.

(7) Present one table that for each year gives the metrics

$$\omega^{\nu}_{st} = \frac{\nu_{\text{as}}}{\nu_{\text{st}}}\,, \qquad \omega^{\nu}_{ct} = \frac{\nu_{\text{as}}}{\nu_{\text{ct}}}\,,$$

for each of the three efficient frontiers graphed in part 5. Comment on the implications of what you find.

(8) For the i^{th} asset in groups (A), (B), and (C) combined give ρ_i^{mn} and ρ_i^{mx} where

$$\rho_i^{\text{mn}} = \min\{\rho_i(d) \,:\, d = 1, \cdots, D\}\,, \qquad \rho_i^{\text{mx}} = \max\{\rho_i(d) \,:\, d = 1, \cdots, D\}\,.$$

Comment on the significance of these numbers.

(9) Determine where along the frontier, efficient frontier, and efficient long frontiers graphed in parts 5 and 6 are the portfolio allocations in Ω_0, $\Omega_{\frac{1}{2}}$, $\Omega_{\frac{3}{4}}$, $\Omega_{\frac{7}{8}}$. (This requires you to devise an algorithm to do this!) Indicate these on the these graphs, say by using different colors, or by using markers. Comment on the implications of what you find.