AMSC/MATH 420, Spring 2018
Second Solo Homework:
Introduction to the Thread

Due Thursday, February 8

Exercise 1. Compute m_i, v_{ij}, and c_{ij} for each of the following groups of assests based on adjusted daily closing price data with uniform weights:

(i) VITSX, VFIUX, and VGSLX in 2017.

(ii) VITSX, VFIUX, and VGSLX in 2016 and 2017.

(iii) VITSX, VFIUX, VGSLX, Apple, Exxon-Mobil, and UPS in 2017.

(iv) VITSX, VFIUX, VGSLX, Apple, Exxon-Mobil, and UPS in 2016 and 2017.

a. Describe the assets VITSX, VFIUX, and VGSLX. Display m_i as a 3-vector and v_{ij} and c_{ij} as 3×3-matrices for groups (i) and (ii). Explain the differences between these objects for groups (i) and (ii).

b. Compute a complete set of eigenpairs of the 3×3-matrices $\{v_{ij}\}$ for groups (i) and (ii). What conclusions do you draw from these?

c. Display m_i as a 6-vector and v_{ij} and c_{ij} as 6×6-matrices for groups (iii) and (iv). Explain the differences between these objects for groups (iii) and (iv).

d. Compute a complete set of eigenpairs for the 6×6-matrices $\{v_{ij}\}$ for groups (iii) and (iv). What conclusions do you draw from these?

e. Give short explanations for the values of c_{ij} that you computed for groups (iii) and (iv).

Exercise 2. Consider the database BERNARD & KILLWORTH fraternity at:
http://vlado.fmf.uni-lj.si/pub/networks/data/ucinet/ucidata.htm
The datafile bkfrat.dat includes two 58×58 matrices. Use the symmetric matrix BKFRAB.

To answer the following questions you need to write a piece of Matlab code that loads relevant parts ofthe bkfrat.dat file and formats the data into a 58x58 matrix. Denote this matrix by W. Make sure you load data correctly. W must be a symmetric matrix. Its first line should read like this:

0 0 2 1 0 0 2 0 0 0 1 1 2 0 0 0 1 0 1 0 0 1 0 0 0 0 0 0 2 1 1 1 0 2 1 2 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 1 1 4 1 1

a. Write code that computes the weighted graph Laplacian $\Delta = D - W$, the normalized symmetric weighted graph Laplacian $\tilde{\Delta} = D^{-1/2}\Delta D^{-1/2}$, the normalized asymmetric weighted graph Laplacian $L = D^{-1}\Delta$.

b. Compute the eigenvalues of the three Laplacian matrices computed earlier, order them ascendingly, and plot these eigenvalues in three separate plots.

c. Find if the weighted graph is connected or not. If not, determine the number of connected components.

d. Set the threshold to $\tau = 20$. Construct a new graph over the set of $n = 58$ vertices as follows: Vertices i and j are connected by an edge if an only if $W_{i,j} \geq \tau$. Denote A_τ the ajacency matrix of this graph. Determine if the graph is connected, and compute the number of connected components. Let $N_c(\tau)$ denote this number.

e. Repeat part e) for $\tau = 19, 18, 17, 16, 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0$. In each case determine the number of connected components, $N_c(\tau)$ and then plot N_c as function of τ. What is the largest value of τ so tha graph is still connected?