

Math 420, Spring 2018

Third Team Homework

Risk-Free Assets and Efficient Frontiers

due Tuesday, 6 March, 2018

Exercise 1. For each of the years ending December 31 of 2013-2017 use one-year histories with uniform weights to compute \mathbf{m} and \mathbf{V} for the risky assets in group (A), in groups (A) and (B) combined, and in groups (A), (B), and (C) combined. (This is the same exercise as the last two times!)

Exercise 2. Assume that the safe investment for each year is the U.S. T-Bill rate available at the beginning of that year. Assume that the credit-line for each year is three points higher than the U.S. T-Bill rate. For each \mathbf{m} and \mathbf{V} computed in Exercise 1 compute the tangency portfolio allocations \mathbf{f}_{st} and \mathbf{f}_{ct} whenever they exist. Present these in three tables (one for group (A), one for groups (A) and (B) combined, and one for groups (A), (B), and (C) combined) that lists years and the allocations for each asset rounded to the nearest hundred thousandth. Identify when each of these portfolios exists and when it does determine if it is long or solvent. Comment on the implications of what you find.

Exercise 3. For each year and each of the three collections of assets (A, AB, ABC) graph in the $\sigma\mu$ -plane:

- the volatility and return mean for that year of each risky and risk-free asset;
- the volatility and return mean for that year of the Markowitz portfolio with allocation that is equidistributed within the collection of assets;
- the volatility and return mean for that year of the minimum volatility portfolio;
- the unlimited efficient frontier (leverage limit $\ell = \infty$) that is associated with the appropriate \mathbf{m} and \mathbf{V} were computed in Exercise 1 and with the risk-free assets from Exercise 2;
- the long efficient frontier (leverage limit $\ell = 0$) that is associated with the appropriate \mathbf{m} and \mathbf{V} were computed in Exercise 1 and with the safe investment from Exercise 2;
- the limited efficient frontier for leverage limit $\ell = 1$ that is associated with the appropriate \mathbf{m} and \mathbf{V} were computed in Exercise 1 and with the risk-free assets from Exercise 2;
- the limited efficient frontier for leverage limit $\ell = 5$ that is associated with the appropriate \mathbf{m} and \mathbf{V} were computed in Exercise 1 and with the risk-free assets from Exercise 2.

There should be 15 graphs (one for each year and each collection of assets) each with eleven assets, two portfolios, four efficient frontiers, and all the tangent portfolios of the four efficient frontiers plotted. Use different symbols or colors to distinguish points associated with the different groups (A), (B), and (C). Comment on any relationships that you see between the objects plotted on each graph. (This will be easier to do if you use the same scales for each of the graphs. Each σ -axis should begin at $\sigma = 0$.)