# Math 420, Spring 2018
## Second Team Homework
## Markowitz Frontiers
due Tuesday, 27 February, 2018

**Exercise 1.** For each of the years ending December 31 of 2013-2017 use one-year histories with uniform weights to compute $\mathbf{m}$ and $\mathbf{V}$ for the risky assets in group (A), in groups (A) and (B) combined, and in groups (A), (B), and (C) combined. (This is the same exercise as last time!) For each $\mathbf{m}$ compute

$$\mu_{\mathrm{mn}} = \min_i\{m_i\}\,, \qquad \mu_{\mathrm{mx}} = \max_i\{m_i\}\,.$$

**Exercise 2.** For each $\mathbf{m}$ and $\mathbf{V}$ computed in Exercise 1 compute the minimum volatility portfolio allocation $\mathbf{f}_{\mathrm{mv}}$. Present these in three tables (one for group (A), one for groups (A) and (B) combined, and one for groups (A), (B), and (C) combined) that lists years and the allocations for each asset rounded to the nearest hundred thousandth. Determine if each of these portfolios is long or solvent. Comment on how these change from year to year for the same groupings of assets.

**Exercise 3.** For each year and each of the three collections of assets (A, AB, ABC) graph in the $\sigma\mu$-plane:
- the volatility and return mean for that year of each asset;
- the volatility and return mean for that year of the Markowitz portfolio with allocation that is equidistributed within the collection of assets;
- the volatility and return mean for that year of the minimum volatility portfolio;
- the unlimited frontier (leverage limit $\ell = \infty$) that is associated with the appropriate $\mathbf{m}$ and $\mathbf{V}$ were computed in Exercise 1;
- the long frontier (leverage limit $\ell = 0$) that is associated with the appropriate $\mathbf{m}$ and $\mathbf{V}$ were computed in Exercise 1;
- the limited frontier for leverage limit $\ell = 1$ that is associated with the appropriate $\mathbf{m}$ and $\mathbf{V}$ were computed in Exercise 1;
- the limited frontier for leverage limit $\ell = 5$ that is associated with the appropriate $\mathbf{m}$ and $\mathbf{V}$ were computed in Exercise 1.

There should be 15 graphs (one for each year and each collection of assets) each with nine assets, two portfolios, and four frontiers plotted. Recall the limited frontier for leverage limit $\ell$ only exists for $\mu \in [\mu_{\mathrm{mn}}(\ell), \mu_{\mathrm{mx}}(\ell)]$ where

$$\mu_{\mathrm{mn}}(\ell) = \mu_{\mathrm{mn}} - \ell(\mu_{\mathrm{mx}} - \mu_{\mathrm{mn}})\,, \qquad \mu_{\mathrm{mx}}(\ell) = \mu_{\mathrm{mx}} + \ell(\mu_{\mathrm{mx}} - \mu_{\mathrm{mn}})\,.$$

Use different symbols or colors to distinguish points associated with the different groups (A), (B), and (C). Comment on any relationships that you see between the objects plotted on each graph. (This will be easier to do if you use the same scales for each of the graphs. Each $\sigma$-axis should begin at $\sigma = 0$.)