

# Math 420, Spring 2018

## First Team Homework

### Risky Assets and Markowitz Portfolios

due Tuesday, 20 February, 2018

**Exercise 1.** Describe each asset in groups (A), (B), and (C). If it is a fund then describe the assets held by the fund. If it is a company then describe the company. (This information is easily found on the internet. A short paragraph on each is all that you need.)

**Exercise 2.** For each asset  $i$  in groups (A), (B), and (C) compute the value-ratio histories  $\{\rho_i(d)\}_{d=1}^D$  for the five years 2013-2017. Also compute the value-ratio history  $\{\rho_{\text{eq}}(d)\}_{d=1}^D$  for the Markowitz portfolio with equal allocations in each asset for the five years 2013-2017. Recall that for every  $d = 1, \dots, D$  we have

$$\rho_{\text{eq}}(d) = \frac{1}{9} \sum_{i=1}^9 \rho_i(d).$$

Plot each history as a function of  $d$ . (There are ten histories, each with about 1260 entries.) Comment on what you find.

**Exercise 3.** For each of your nine assets compute

$$\begin{aligned} \rho_i^{\text{mn}} &= \min\{\rho_i(d) : d = 1, \dots, D\}, \\ \rho_i^{\text{mx}} &= \max\{\rho_i(d) : d = 1, \dots, D\}. \end{aligned}$$

For the Markowitz portfolio with equal allocations in each asset compute

$$\begin{aligned} \rho_{\text{eq}}^{\text{mn}} &= \min\{\rho_{\text{eq}}(d) : d = 1, \dots, D\}, \\ \rho_{\text{eq}}^{\text{mx}} &= \max\{\rho_{\text{eq}}(d) : d = 1, \dots, D\}. \end{aligned}$$

Order these ten assets from smallest to largest  $\rho^{\text{mx}}/\rho^{\text{mn}}$ . Comment on what you find.

**Exercise 4.** For the assets in group (A), in groups (A) and (B) combined, and in groups (A), (B), and (C) combined compute

$$\begin{aligned} \rho_{\text{mn}}(d) &= \min\{\rho_i(d) : i = 1, \dots, N\}, \\ \rho_{\text{mx}}(d) &= \max\{\rho_i(d) : i = 1, \dots, N\}, \\ \ell_0(d) &= \frac{\rho_{\text{mn}}(d)}{\rho_{\text{mx}}(d) - \rho_{\text{mn}}(d)}. \end{aligned}$$

Plot these as a function of  $d$ . (There will be nine plots.) Comment on what you find. (Recall that  $\ell_0(d)$  is the leverage limit above which some Markowitz portfolios become insolvent.)

**Exercise 5.** For each of the years ending December 31 of 2013-2017 use one-year histories with uniform weights to compute  $\mathbf{m}$  and  $\mathbf{V}$  for the risky assets in group (A), in groups (A) and (B) combined, and in groups (A), (B), and (C) combined. (There will be fifteen vectors  $\mathbf{m}$  and matrices  $\mathbf{V}$ , three for each of the five years.) For each  $\mathbf{V}$  compute its eigenpairs. Order the eigenpairs from largest eigenvalue to smallest. Comment on what you find.