Math 420, Spring 2018
Seventh Team Homework

due Tuesday, 24 April, 2018

Consider the dataset you chose for your project. Consider the Laplacian Eigenmap algorithm that determines a geometric graph from a set of weights.

Let n denote the size of your graph (number of nodes), and let d denote a running integer. Write a Matlab script that performs the following tasks:

1. Compute the diagonal matrix D, the weighted Laplacian $\Delta = D - W$ and the normalized weighted Laplacian $\tilde{\Delta} = D^{-1/2}\Delta D^{-1/2}$.

2. Solve the Eigen-problems:

$$\tilde{\Delta} e_k = \lambda_k e_k \quad , \quad \|e_k\| = 1$$

and sort the eigenvalues ascendingly, $\lambda_1 \leq \lambda_1 \leq \cdots \leq \lambda_n$. You will need only the bottom $d+1$ eigenpairs.

3. Compute the generalized eigenvectors $f_k = D^{-1/2} e_k$, and construct the $d \times n$ matrix

$$Y = \begin{bmatrix} f_2^T \\ \vdots \\ f_{d+1}^T \end{bmatrix}.$$

4. Retrieve the geometric graph points $\{y_1, \ldots, y_n\}$ as the n columns of Y.

5. Compute the criterion

$$J(y_1, \ldots, y_n) = \sum_{1 \leq i,j \leq n} W_{i,j} \|y_i - y_j\|^2$$

A. Run you code for $d = 1, 2, 3, \ldots, n$ on your dataset (WeightData.txt) and plot the value of criterion J obtained at step 5, as function of d.

B. For $d = 2$ plot the geometric graph.

Your submission should include: 1. Matlab script code that solves the above tasks; 2. Two figures: one for $J = J(d)$ and the other for the geometric graph for $d = 2$.