

**Math 420, Spring 2018**  
**Fifth Team Homework**  
due Thursday, 12 April, 2018

Consider the datafile WeightData.txt that contains a weight matrix  $W$  as described below. The weights have been computed using the exponential formula  $W_{i,j} = e^{-\|y_i - y_j\|^2}$ . In this problem you need to estimate the appropriate dimension  $d$  for an exact embedding of the data set. Write a (Matlab) code that performs the following tasks:

- a. Loads the matrix  $W$ ;
- b. Computes the matrix of pairwise distances  $S = (S_{i,j})_{1 \leq i,j \leq n}$  based on the exponential model indicated above.
- c. Compute the Gramm matrix of the centered set of points,  $G$ .
- d. Using the SVD decomposition of  $G$  find the set of singular values (eigenvalues) and determine the minimal isometric embedding of  $S$ .
- e. Compute the coordinates of a set of  $n$  vectors  $y_1, y_2, \dots, y_n$  so that  $\sum_{k=1}^n y_k = 0$  and  $\|y_i - y_j\|^2 = S_{i,j}$ ,  $1 \leq i, j \leq n$ .
- f. Visualize the  $n$  points by projecting onto the space spanned by the first two coordinates, i.e. plot the 2-D points  $(y_{1,1}, y_{1,2}), (y_{2,1}, y_{2,2}), \dots, (y_{n,1}, y_{n,2})$ .

**Note** The data file of weights has the following format:

First line:  $n$

Second line:  $W_{11} W_{12} \dots W_{1n}$

Third line:  $W_{21} W_{22} \dots W_{2n}$

...

Last line (the  $n+1$ st):  $W_{n1} W_{n2} \dots W_{nn}$

In other words: the first line includes the number of vertices  $n$ ; the second line contains  $n$  floats, the first row of  $W$ ; the other lines contain the other rows of  $W$ ; the last line of the file includes the last row of  $W$ .