Math 420, Spring 2018
Third Team Homework

due Tuesday, 6 March, 2018

Exercise 1. [4pts] Write a function that computes the second smallest eigenvalue λ_1 of the normalized graph Laplacian for a given graph. Then write a script that uses the same dateset you used last time, and computes the sequence of second smallest eigenvalue $\lambda_1(k)$ of the cumulative graph $Edges(1:k, 1:2)$, where $1 \leq k \leq m$ denotes the running number of edges. Run the script on graph.dat and print out all the codes (function and script) and results. For interoperability between homeworks, create a data file, say graph.dat , that has the following format:

```
First line: n m
Second line: Edge1Vertex1 Edge1Vertex2
Third line: Edge2Vertex1 Edge2Vertex2
...
m+1st line: EdgemVertex1 EdgemVertex2
```

Exercise 2. [6pts] Consider the weighted undirected graph inserted below.

1. [2pts] Write down the weight matrix W, the weighted graph Laplacian $\Delta = D - W$, and the normalized weighted graph Laplacian $\tilde{\Delta}$. Compute its eigenvalues and eigenvectors.

2. [2pts] Write a function that computes the Cheeger constant and the optimal partition for a given weight matrix W, and apply it to this graph. Determine both the optimal partition and the Cheeger constant h_G.

3. [2pts] Use the second smallest eigenpair obtained at the first part to determine an alternate partition (what we called in class the "initialization"). Find the value of the criterion minimized by the Cheeger constant and compare it to h_G.

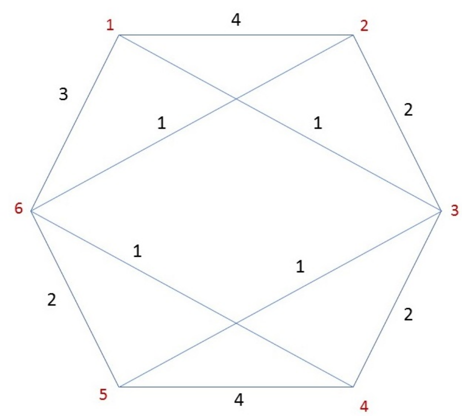

Figure 1: A weighted graph with $n = 6$ vertices. The vertex labels are marked in red. The edge weights are in black.