

Math 420, Spring 2018
First Team Homework
due Tuesday, 27 February 2018

Exercise 1. Consider an Erdős-Rényi random graph G with $n = 100$ vertices and probability $p = 0.9$ for each edge, i.e. $G \in \mathcal{G}_{100,0.9}$.

1. (1pt) What is the expected number of edges ?
2. (1pt) For each vertex v , the degree $deg(v)$ is defined as the number of edges that have v as one of end points. (Thus in a complete graph with n vertices, each vertex has degree $n - 1$). For the random graph G , compute the expected degree of each vertex.
3. (1pt) Assume each edge of G is colored either in red, or in blue. Given an edge, assume the probability of being red is 30% whereas the probability of being blue is 70%. Determine the expected numbers of red edges and of blue edges.
4. (1pt) Determine the expected number of 3-cliques.
5. (1pt) Determine the expected number of 4-cliques.

Exercise 2. (5pts) Write a Matlab script (call it teamXhw1.m) and a Matlab function (call it cliques.m) that perform the following steps:

1. [2pts] The Matlab function cliques.m should have the following preamble:
function qp = cliques(E,n,m,p)

```
% CLIQUES computes cumulative count of cliques of a given ordered graph.
% Use:
% qp = cliques(E,n,m,3)
% Inputs:
% E is a mx2 matrix of ordered edges (matrix of integers).
% n is the number of vertices (integer)
% m is the number of edges (integer)
% p is the clique order (integer); for now p should be set to 3.
% Outputs:
% qp is a m-vector of integers. qp(k) should contain the number of p-cliques
% of the graph whose set of edges is E(1:k,1:2)
%
```

The function should compute the cumulative count of 3-cliques of a given graph. Its inputs are:

1. the ordered sequence of unoriented edges $E(1 : m, 1 : 2)$, where the k^{th} edge is between vertex $E(k, 1)$ and $E(k, 2)$;
2. the number of vertices n ;
3. the number of edges m (somewhat redundant, since it can be read from $size(E, 1)$);

4. the clique order p , that should be set to 3 for now. (In the subsequent homework you will be asked to implement the case $p = 4$).

Its output is a length m vector of integers, call it qp , that has the following significance: $qp(k)$ is the number of p -cliques (3-cliques in this homework) in the graph over n vertices that has k edges, namely $E(1 : k, 1 : 2)$, that is, $\{(E(1, 1), E(1, 2)), (E(2, 1), E(2, 2)), (E(3, 1), E(3, 2)), \dots, (E(k, 1), E(k, 2))\}$.

In this homework the function `cliques()` should work for $p = 3$.

2. The Matlab script `teamXhw1.m` should do the following:

- 1pt Load data file and turns into an ordered sequence of edges. If the raw data is a weight matrix (such as `BKFRAT`), then use the weights to order the edges (from highest to smallest weight); if data set is sequence of links (such as in the `ca-***` arXiv preprint list server), then use that sequence of links as the ordered sequence of edges. For testing purposes use `BKFRAT` until you get a different dataset to work with.
- 1pt Call `cliques()` Matlab function you wrote at part 1) to construct the sequence of 3-cliques on this ordered edges.
- 1pt Plot the cumulative count of 3-cliques $qp(1 : m)$ against the number of edges $1 : m$.