Math 420, Spring 2017
Second Project: Calibration Uncertainty and Risk

presentation due Thursday, 4 May, 2017
report due Thursday, 11 May, 2017

This project explores how to use calibration uncertainty to guide the choice of the risk coefficient χ. Consider the following groups of assets.

(A) This will be the Group A from the first project.
(B) This will be the Group B from the first project of one of the team members. It will be filled in once the team is assigned.
(C) This will be the Group C from the first project of one of the team members. It will be filled in once the team is assigned.

For each of the years ending December 31 of the years 2002-2016 use one-year histories of daily returns and uniform weights to calabrate $\mathbf{m}$ and $\mathbf{V}$. Also use two-year histories of daily return rates and uniform weights to calabrate $\mathbf{m}_{\mathrm{T}}$ and $\mathbf{V}_{\mathrm{T}}$.

For every $s \in [0, 1]$ define

$$\mathbf{m}(s) = (1 - s)\mathbf{m} + s\mathbf{m}_{\mathrm{T}}\,, \qquad \mathbf{V}(s) = (1 - s)\mathbf{V} + s\mathbf{V}_{\mathrm{T}}\,.$$

Show that $\mathbf{m}(s)$ and $\mathbf{V}(s)$ are calibrations that use two-year histories of daily return rates and nonuniform weights and identify these nonuniform weights as a function of s. The question to be investigated is "How sensitive is the optimal growth rate γ as a function of s for a given risk aversion χ?"

Show that the optimal growth rate γ is a differentiable function of s, and derive a formula for $\partial_s\gamma$. Use this formula evaluated at $s = 0$ to devise at least four measures of this sensitivity.

Repeat the last homework assignment with $\chi = 0$, .25, .5, .75, 1, 1.25, 1.5, 1.75 and 2. Determine which value of χ yields the best performing portfolios in the subsequent year. Use scatter plots to seek correlations between these best χ and the measures that you devised above. Identify the two measures with the strongest correlation and find a linear function of those measures that best fits these χ.