Math 420, Spring 2017
First Project: Rolling Frontiers

presentation Tuesday, 14 March, 2017
report due Thursday, 16 March, 2017

This project explores how efficient frontiers evolve over time. Consider the following groups of assets.

(A) VITSX, VFIUX, VGSLX.
(B) from the "Projects" webpage.
(C) Apple, Berkshire Hathaway, General Electric.

(1) Identify the funds in (A) and (B) and describe their holdings. (This information should inform some of your subsequent answers.)

(2) For each of the years ending December 31 of 2002-2016 use one-year histories with uniform weights to compute $\mathbf{m}$ and $\mathbf{V}$ for the risky assets in group (A), in groups (A) and (B) combined, and in groups (A), (B), and (C) combined.

(3) For each $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 compute the minimum volatility portfolio allocation $\mathbf{f}_{\text{mv}}$. Present these in three tables (one for group (A), one for groups (A) and (B) combined, and one for groups (A), (B), and (C) combined) that lists years and the allocations for each asset rounded to the nearest thousandth. Determine if each of these portfolios is long or solvent. Comment on the implications of what you find.

(4) Assume that the safe investment for each year is the U.S. T-Bill rate available at the beginning of that year. Assume that the credit-line for each year is three points higher than the U.S. T-Bill rate. For each $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 compute the tangency portfolio allocations $\mathbf{f}_{\text{st}}$ and $\mathbf{f}_{\text{ct}}$ whenever they exist. Present these in three tables as was done in part 3. Identify when each of these portfolios exists and when it does determine if it is long or solvent. Comment on the implications of what you find.

(5) For each year graph in the $\sigma\mu$-plane the three frontiers associated with the appropriate $\mathbf{m}$ and $\mathbf{V}$ computed in part 2 and the three efficient frontiers associated with the tangency portfolios found in part 4. Comment on the implications of what you see.

(6) In a similar manner, for each year graph the efficient long frontiers, for the risky assets in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined, with a safe investment of U.S. T-Bills. Comment on the implications of what you see.

(7) Present one table that for each year gives the metrics

$$\omega^{\nu}_{st} = \frac{\nu_{\text{as}}}{\nu_{\text{st}}}\,, \qquad \omega^{\nu}_{ct} = \frac{\nu_{\text{as}}}{\nu_{\text{ct}}}\,,$$

for each of the three efficient frontiers graphed in part 5. Comment on the implications of what you find.

(8) Show how the frontiers in the $\sigma\mu$-plane for the risky assets in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined evolve in time using one-quarter histories with uniform weights. Do this for the year ending December 31 of 2002 and every quarter thereafter until December 31 of 2016 — i.e. for the year ending March 31 of 2002, the year ending June 30 of 2002, and so forth for every quarter until December 31 of 2016. Show these results as a slide-show or better as a slow-frame movie. Do the same for a bar-graph of $\mathbf{f}_{\mathrm{mv}}$. Comment on the implications of what you see. (If you have automated things well then you can try showing these evolutions with a frame every month or every week.)

(9) Similarly show the evolution of the efficient frontier associated with the risk-free assets and the evolution of the tangency portfolios $\mathbf{f}_{\mathrm{st}}$ and $\mathbf{f}_{\mathrm{ct}}$. Comment on the implications of what you see.

(10) Similarly show the evolution of the efficient long frontier, for the risky assets in group (A), groups (A) and (B) combined, and groups (A), (B), and (C) combined, with a safe investment of U.S. T-Bills. Comment on the implications of what you see.