AMSC/MATH 420, Spring 2017
Second Solo Homework:
Introduction to the Thread

Due Thursday, February 9

Exercise 1. Compute m_i, v_{ij}, and c_{ij} for each of the following groups of assests based on adjusted daily closing price data with uniform weights:

(i) VITSX, VFIUX, and VGSLX in 2016.

(ii) VITSX, VFIUX, and VGSLX in 2015 and 2016.

(iii) VITSX, VFIUX, VGSLX, Apple, Exxon-Mobil, and UPS in 2016.

(iv) VITSX, VFIUX, VGSLX, Apple, Exxon-Mobil, and UPS in 2015 and 2016.

a. Describe the assets VITSX, VFIUX, and VGSLX. Display m_i as a 3-vector and v_{ij} and c_{ij} as 3×3-matrices for groups (i) and (ii). Explain the differences between these objects for groups (i) and (ii).

b. Compute a complete set of eigenpairs of the 3×3-matrices $\{v_{ij}\}$ for groups (i) and (ii). What conclusions do you draw from these?

c. Display m_i as a 6-vector and v_{ij} and c_{ij} as 6×6-matrices for groups (iii) and (iv). Explain the differences between these objects for groups (iii) and (iv).

d. Compute a complete set of eigenpairs for the 6×6-matrices $\{v_{ij}\}$ for groups (iii) and (iv). What conclusions do you draw from these?

e. Give short explanations for the values of c_{ij} that you computed for groups (iii) and (iv).

Exercise 2. Consider the three undirected hexagonal graphs shown in Figures 1, 2, and 3. They are each built from a hexagon, with a center added for the graph in Figure 2.

a. For each graph find the number of vertices n, the number of edges m, and write down the list of vertices $\mathcal{V}$ and the list of edges $\mathcal{E}$.

b. For each graph compute the graph Laplacian Δ, the normalized graph Laplacian $\tilde{\Delta}$ and the normalized asymmetric Laplacian $\tilde{L}$.

c. For each graph compute a complete set of eigenpairs for each of the Laplacian matrices Δ, $\tilde{\Delta}$, and $\tilde{L}$.

d. Explain how the symmetry of the graphs is reflected in the spectra of the matrices that was computed in part c.

e. Repeat parts b and c for the analogous graphs built from a regular polygon with 100 vertices. Can you guess the spectrum of the Laplacian matrices in the general case of n vertices?

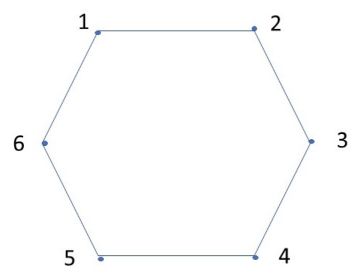

Figure 1: A Hexagonal Graph

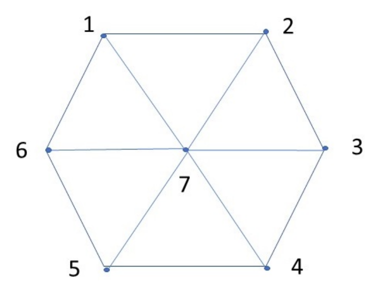

Figure 2: A Hexagonal Graph plus Star

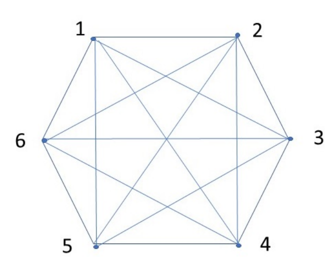

Figure 3: A Hexagonal Complete Graph