Math 420, Spring 2017
Fourth Team Homework

due Thursday, 13 April, 2017

Exercise 1.

Select an online database that involves a weighted undirected graph of at least 100 vertices.

a. Describe the database and the network involved.

b. Create the weighted graph, specifically the weighted matrix W. Save it into a text file, WeightData.txt with the following format:

First line: n

Second line: W11 W12 ... W1n

Third line: W21 W22 ... W2n

...

Last line (the n+1st): Wn1 Wn2 ... Wnn

In other words: the first line includes the number of vertices n; the second line contains n floats, the first row of W; the other lines contain the other rows of W; the last line of the file includes the last row of W.

Exercise 2.

Consider the weight matrix W given in the datafile W.txt The weightes have been computed using the exponential formula $W_{i,j} = e^{-\|y_i - y_j\|^2}$. In this problem you need to estimate the appropriate dimension d for an exact embedding of this data set. Write a (Matlab) code that performs the following tasks:

a. Loads the matrix W;

b. Computes the matrix of pairwise distances $S = (S_{i,j})_{1 \le i,j \le n}$ based on the exponential model indicated above.

c. Compute the Gramm matrix of the centered set of points, G.

d. Using the SVD decomposition of G find the set of singular values (eigenvalues) and determine the minimal isometric embedding of S.

e. Compute the coordinates of a set of n vectors y_1, y_2, ..., y_n so that $\sum_{k=1}^{n} y_k = 0$ and $\|y_i - y_j\|^2 = S_{i,j}$, $1 \le i, j \le n$.

f. Visualize the n points by projecting onto the space spanned by the first two coordinates, i.e. plot the 2-D points $(y_{1,1}, y_{1,2}), (y_{2,1}, y_{2,2}), \ldots, (y_{n,1}, y_{n,2})$.