

**Math 420, Spring 2017**  
**First Team Homework**  
due Thursday, 23 February, 2017

**Exercise 1.** Consider an Erdős-Rényi random graph  $G$  with  $n = 100$  vertices and probability  $p = 0.9$  for each edge, i.e.  $G \in \mathcal{G}_{100,0.9}$ .

1. (1pt) What is the expected number of edges ?
2. (1pt) For each vertex  $v$ , the degree  $deg(v)$  is defined as the number of edges that have  $v$  as one of end points. (Thus in a complete graph with  $n$  vertices, each vertex has degree  $n - 1$ ). For the random graph  $G$ , compute the expected degree of each vertex.
3. (1pt) Assume each edge of  $G$  is colored either in red, or in blue. Given an edge, assume the probability of being red is 30% whereas the probability of being blue is 70%. Determine the expected numbers of red edges and of blue edges.
4. (1pt) Determine the expected number of 3-cliques.
5. (1pt) Determine the expected number of 4-cliques.

**Exercise 2.** (5pts) Write a code (preferably in Matlab, or in your favorite implementation language) that computes the number of 3-cliques of a given graph. The function will have the following format:

**Inputs:**

1.  $n$  : integer, the number of vertices
2.  $m$ : integer, the number of edges
3.  $A$ : the  $n \times n$  adjacency matrix

**Output:**

1.  $N_3$ : integer, the number of 3-cliques