Two-Group Model with Interventions

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AMSC/MATH 420, Spring 2015
Types of Interventions

- We’ll consider two hypothetical types of interventions that public health officials might undertake when the outbreak of an epidemic such as HIV/AIDS is detected.

- The first type, which we’ll call “type A”, reduces or modifies infection-risking behavior among both susceptible and infectious individuals. This might be a public awareness/education campaign, or a program that gives away prophylactics or clean needles.

- The second type, which we’ll call “type B”, only changes the behavior or infectiousness of infectious individuals. This might be a testing program, which allows infectious individuals to be aware of their status sooner, or a treatment program that reduces their infectiousness.
Two-Group Model

- For the two-group SI model, we’ll model the type A and B interventions as follows:

\[
\begin{align*}
\frac{dS_1}{dt} &= -p_{11} S_1 I_1 - p_{12} S_1 I_2 - a_1 S_1 \\
\frac{dI_1}{dt} &= p_{11} S_1 I_1 + p_{12} S_1 I_2 - (a_1 + b_1)I_1 \\
\frac{dS_2}{dt} &= -p_{21} S_2 I_1 - p_{22} S_2 I_2 - a_2 S_2 \\
\frac{dI_2}{dt} &= p_{21} S_2 I_1 + p_{22} S_2 I_2 - (a_2 + b_2)I_2.
\end{align*}
\]

- This model treats removals like the SIR model does, except that it allows people to be removed from the susceptible population without being infected.

- The model allows the interventions to have different effects on the group 1 and group 2 populations.
Simplification Used in Model

• Rather than try to model the reduced likelihood of infection among people who respond to the interventions, we pretend that the interventions cause a certain fraction of people per unit time to be removed from the susceptible or infectious population.

• For example, if a certain number of people reduce their risky behavior by $\frac{1}{2}$, we model this as half of those people eliminating their risky behavior completely – these people are removed.
Accounting for the Size of the Epidemic

- Let $N(a_1, a_2, b_1, b_2)$ be the number of people infected in the long run.
- If we augment the model with the differential equations
  
  $$
  \frac{dR_1}{dt} = (a_1 + b_1)I_1 \\
  \frac{dR_2}{dt} = (a_2 + b_2)I_2,
  $$

  then $R_1$ and $R_2$ represent people from group 1 and group 2 respectively that have been removed from the infected population (but not those removed directly from the susceptible population).
- Then $I_1(T) + R_1(T) + I_2(T) + R_2(T)$ is the cumulative number of people infected up to time $T$, and we can express $N(a_1, a_2, b_1, b_2) = \lim_{T \to \infty}[I_1(T) + R_1(T) + I_2(T) + R_2(T)]$. 
Impact of Interventions

• Recall that with no interventions/removals ($a_1 = a_2 = b_1 = b_2 = 0$), all susceptibles get infected in the long run:
  $N(0, 0, 0, 0) = S_1(0) + I_1(0) + S_2(0) + I_2(0)$.

• Define the impact of the interventions with parameters $a_1, a_2, b_1, b_2$ to be
  $$M(a_1, a_2, b_1, b_2) = \frac{N(0, 0, 0, 0) - N(a_1, a_2, b_1, b_2)}{N(0, 0, 0, 0)}.$$

• This “impact” is the fraction of people who would have been infected in the no-intervention model that never get infected in the model with interventions.

• Our next goal will be to maximize the impact subject to a constraint on the intervention parameters $a_1, a_2, b_1, b_2$. 