Sample Project

1. **(Epidemiology)** The spread of a communicable disease among a population is often modeled using differential equations. To simplify the model, let's assume that the disease is not fatal and that the total population is fixed.

   (a) Assume that there is a constant "recovery rate" among the infected population, meaning that the number of infected people becoming cured per unit time is proportional to the total number of infected people. Assume that once cured, people are susceptible to being infected again, and that the "infection rate" (the chances of an uninfected person becoming infected per unit time) is proportional to the number of infected people. Construct a model for the evolution of the number of infected people over time, and discuss the underlying assumptions (in terms of the disease and of human behavior) on which this model is based.

   (b) Researchers have obtained the following estimates for the size of the infected population: 10 years ago, 0.2% of the total population; 5 years ago, 0.8%; and now, 3%. What parameters in your model are consistent with these observations? Based on these parameters, how will the size of the infected population evolve in the future? How would your predictions change if the current percentage of people that are infected were instead 2.9%, or 3.1%? Also, what would you do if more or fewer data points were known?

   (c) Assume now that a certain fraction of the population being cured at a given time becomes immune to the disease, and that people who are cured without becoming immune still have the same chance of becoming immune after being reinfected and cured again. How can the model be modified to take immunity into account? Is the new model consistent with the data above, and if so is any additional data needed to predict how the size of the infected population will evolve? What will happen in the long run?

   (d) How could you make the model more realistic by taking into account changes in the population due to factors like births, deaths and immigration/emigration? Qualitatively speaking, how would these factors affect your results?