

# AMSC/MATH 420 Project Two, Spring 2014

## Modeling Portfolios: Calibration Uncertainty

oral presentation due Friday, 2 May, 2014

written report due Monday, 12 May, 2014

This project explores how to use calibration uncertainty to guide the choice of the risk aversion coefficient. Consider the following groups of assets:

- (A) this will be the Group A from the first project;
- (B) this will be the Group B from the first project of one of the team members and will be decided after the team is assigned.

For each of the years ending Decemebr 31 of the years 2008-2013 use one-year histories of daily return rates and uniform weights to calabrate  $\mathbf{m}$  and  $\mathbf{V}$ . Also use two-year histories of daily return rates and uniform weights to calabrate  $\mathbf{m}_T$  and  $\mathbf{V}_T$ .

For every  $s \in [0, 1]$  define

$$\mathbf{m}(s) = (1 - s)\mathbf{m} + s\mathbf{m}_T, \quad \mathbf{V}(s) = (1 - s)\mathbf{V} + s\mathbf{V}_T.$$

Show that  $\mathbf{m}(s)$  and  $\mathbf{V}(s)$  are calibrations that use two-year histories of daily return rates and nonuniform weights and identify these nonuniform weights as a function of  $s$ . The question to be investigated is “How sensitive is the optimal growth rate  $\gamma$  as a function of  $s$  for a given risk aversion  $\chi$ ?”

Show that the optimal growth rate  $\gamma$  is a differentiable function of  $s$ , and derive a formula for  $\partial_s \gamma$ . Use this formula evaluated at  $s = 0$  to devise at least four measures of this sensitivity.

Repeat the last homework assignment with  $\chi = 0, .25, .5, .75, 1, 1.25, 1.5, 1.75$  and  $2$ . Determine which value of  $\chi$  yields the best performing portfolios in the subsequent year. Use scatter plots to seek correlations between these best  $\chi$  and the measures that you devised above. Identify the two measures with the strongest correlation and find a linear function of those measures that best fits these  $\chi$ .