Two-Group Model with Interventions

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Types of Interventions

- We'll consider two hypothetical types of interventions that public health officials might undertake when the outbreak of an epidemic such as HIV/AIDS is detected.
- The first type, which we'll call "type A", reduces or modifies infection-risking behavior among both susceptible and infectious individuals. This might be a public awareness/education campaign, or a program that gives away prophylactics or clean needles.
- The second type, which we'll call "type B", only changes the behavior or infectuousness of infectious individuals. This might be a testing program, which allows infectious individuals to be aware of their status sooner, or a treatment program that reduces their infectuousness.

Two-Group Model

- For the two-group SI model, we'll model the type A and B interventions as follows:

$$
\begin{aligned}
dS_1/dt &= -p_{11}S_1\mathcal{I}_1 - p_{12}S_1\mathcal{I}_2 - a_1S_1 \\
d\mathcal{I}_1/dt &= p_{11}S_1\mathcal{I}_1 + p_{12}S_1\mathcal{I}_2 - (a_1 + b_1)\mathcal{I}_1 \\
dS_2/dt &= -p_{21}S_2\mathcal{I}_1 - p_{22}S_2\mathcal{I}_2 - a_2S_2 \\
d\mathcal{I}_2/dt &= p_{21}S_2\mathcal{I}_1 + p_{22}S_2\mathcal{I}_2 - (a_2 + b_2)\mathcal{I}_2.
\end{aligned}
$$

- This model treats removals like the SIR model does, except that it allows people to be removed from the susceptible population without being infected.
- The model allows the interventions to have different effects on the group 1 and group 2 populations.

Simplification Used in Model

- Rather than try to model the reduced likelihood of infection among people who respond to the interventions, we pretend that the interventions cause a certain fraction of people per unit time to be removed from the susceptible or infectuous population.
- For example, if a certain number of people reduce their risky behavior by $1/2$, we model this as half of those people eliminating their risky behavior completely – these people are removed.

Accounting for the Size of the Epidemic

- Let $N(a_1, a_2, b_1, b_2)$ be the number of people infected in the long run.
- If we augment the model with the differential equations

$$dR_1/dt = (a_1 + b_1)\mathcal{I}_1$$
$$dR_2/dt = (a_2 + b_2)\mathcal{I}_2,$$

  then R_1 and R_2 represent people from group 1 and group 2 respectively that have been removed from the infected population (but not those removed directly from the susceptible population.
- Then $\mathcal{I}_1(T) + R_1(T) + \mathcal{I}_2(T) + R_2(T)$ is the cumulative number of people infected up to time T, and we can express $N(a_1, a_2, b_1, b_2) = \lim_{T\to\infty}[\mathcal{I}_1(T) + R_1(T) + \mathcal{I}_2(T) + R_2(T)]$.

Impact of Interventions

- Recall that with no interventions/removals ($a_1 = a_2 = b_1 = b_2 = 0$), all susceptibles get infected in the long run:
  $N(0,0,0,0) = S_1(0) + \mathcal{I}_1(0) + S_2(0) + \mathcal{I}_2(0)$.
- Define the impact of the interventions with parameters a_1, a_2, b_1, b_2 to be

$$M(a_1, a_2, b_1, b_2) = \frac{N(0,0,0,0) - N(a_1, a_2, b_1, b_2)}{N(0,0,0,0)}.$$

- This “impact” is the fraction of people who would have been infected in the no-intervention model that never get infected in the model with interventions.
- Our next goal will be to maximize the impact subject to a constraint on the intervention parameters a_1, a_2, b_1, b_2.