

AMSC/Math 420, Spring 2014
Fourth Team Homework
Modeling Portfolios: IID Models of Single Assets

Due Monday, 31 March, 2014

In the following exercises consider the risky assets in groups (A) and (B) of your project.

Exercise 1. For each asset consider its daily return rate history $\{(d, r(d))\}_{d=1}^D$ over the year starting January 1, 2013. For each asset make the three scatter-plots

$$(r(d), r(d+1)) \Big|_{d=1}^{D-1}, \quad (r(d), r(d+5)) \Big|_{d=1}^{D-5}, \quad (r(d), r(d+25)) \Big|_{d=1}^{D-25}.$$

What do these plots say about the temporal correlations in these daily return rate histories? What are the implications for adopting an IID model for these assets?

Exercise 2. For each asset consider its daily return rate history $\{(d, r(d))\}_{d=1}^D$ over the year starting January 1, 2013. For each asset plot its history in the dr -plane. Use these plots to explain when each of these assets is a good candidate to be mimiced by an IID model.

Exercise 3. Consider one-year histories of daily share price data for each asset over the years ending December 31 of 2008-2013. Use the unbiased estimators $\hat{\mu}$, $\hat{\xi}$, $\hat{\gamma}$, and $\hat{\theta}$ given by

$$\hat{\mu} = \frac{1}{D} \sum_{d=1}^D r(d), \quad \hat{\xi} = \frac{1}{D-1} \sum_{d=1}^D (r(d) - \hat{\mu})^2,$$

$$\hat{\gamma} = \frac{1}{D} \sum_{d=1}^D x(d), \quad \hat{\theta} = \frac{1}{D-1} \sum_{d=1}^D (x(d) - \hat{\gamma})^2,$$

to estimate μ , ξ , γ , and θ given the share price history $\{s(d)\}_{d=0}^D$ with

$$r(d) = D \left(\frac{s(d)}{s(d-1)} - 1 \right), \quad x(d) = D \log \left(\frac{s(d)}{s(d-1)} \right).$$

How do $\hat{\mu}$ and $\hat{\gamma}$ compare as measures of reward over the years considered? How do $\hat{\xi}$ and $\hat{\theta}$ compare as measures of risk?

Remark. Notice that the sum defining $\hat{\gamma}$ telescopes, whereby

$$\hat{\gamma} = \log \left(\frac{s(D)}{s(0)} \right).$$