AMSC/MATH 420, Spring 2014
Modeling Epidemics: Team Homework 6

due Monday April 14

We'll continue with the two-group SI model with two types of interventions:

$$\begin{aligned} dS_1/dt &= -p_{11}S_1\mathcal{I}_1 - p_{12}S_1\mathcal{I}_2 - a_1S_1 \\ d\mathcal{I}_1/dt &= p_{11}S_1\mathcal{I}_1 + p_{12}S_1\mathcal{I}_2 - (a_1+b_1)\mathcal{I}_1 \\ dS_2/dt &= -p_{21}S_2\mathcal{I}_1 - p_{22}S_2\mathcal{I}_2 - a_2S_2 \\ d\mathcal{I}_2/dt &= p_{21}S_2\mathcal{I}_1 + p_{22}S_2\mathcal{I}_2 - (a_2+b_2)\mathcal{I}_2. \end{aligned}$$

We're modeling the cost of an intervention parameter quadruple (a_1, a_2, b_1, b_2) to be $K_c(a_1, a_2, b_1, b_2) = ca_1+ca_2+b_1+b_2$, where c is a positive number. For this assignment, set a budget of $K_c(a_1, a_2, b_1, b_2) = 0.04$ and consider the optimal paramaters to be those within the budget that maximize the impact $M(a_1, a_2, b_1, b_2)$. (Note: I'm setting a budget that I think will allow you impacts in a similar range as the previous assignment no matter what c is. If you are getting impacts very close to 0 or very close to 1 for your transmission parameters, try a different budget.)

For each of the two metropolitan areas assigned to your team, use the transmission parameters $p_{11}, p_{12}, p_{21}, p_{22}$ you found by fitting the data and answer the following questions:

1. What is the largest value of c for which the optimal parameters have $b_1 = b_2 = 0$? (One decimal place of accuracy is fine for this and the next question.)

2. What is the smallest value of c for which the optimal parameters have $a_1 = a_2 = 0$?

3. For a range of c values in between the values you found above, determine the optimal a_1, a_2, b_1, b_2 and graph these values as a function of c. Are there values of c for which the optimal parameters are all positive, and/or for which 3 of 4 are positive?

Finally, discuss how you plan to address the questions raised in your multi-week project with each other and with the instructor. Write a paragraph that reflects the outcome of that discussion.