AMSC/MATH 420, Spring 2014
Modeling Epidemics: Team Homework 5

due Monday April 7

To get a sense of how "type a" and "type b" interventions affect the size of an outbreak, let's first consider them in the one-group SI model:

$$dS/dt = -pS\mathcal{I} - aS$$
$$d\mathcal{I}/dt = pS\mathcal{I} - (a+b)\mathcal{I}.$$

As noted in class, one way to account for the total number of people infected (according to the model) is to add a "removed" population $R(t)$ with $R(0) = 0$ and

$$dR/dt = (a+b)\mathcal{I}.$$

Notice that R only counts people removed from the infectuous population, not from the susceptible population. Then $I(T) + R(T)$ is the total number of people infected up to time T. To assess the size of the entire outbreak, you should use a value of T large enough so that $S'(T)$ is close to zero. Remember that if $a = b = 0$, the total number of people infected will eventually be $N = S(0) + I(0)$. And in class, we defined the "impact" M of an intervention to be the fraction of people the intervention prevents from being infected (which is 1 minus the fraction of people who are eventually infected).

1. Choose values of N and p appropriate to one your data sets (the exact values are not so important), and set $I(0) = 0.01N$. Find the approximate value (2 significant digits is fine) of $a_{0.5}$ such that impact is 0.5 when $a = a_{50}$ and $b = 0$; that is, $M(a_{0.5}, 0) = 0.5$. Similarly, find $a_{0.75}$ for which $M(a_{0.75}, 0) = 0.75$. Also, find $b_{0.5}$ and $b_{0.75}$ that achieve impacts of 0.5 and 0.75, respectively, when $a = 0$. Next suppose that you divide your resources between the two interventions in the following sense: let $a = 0.5a_{0.5}$ and $b = 0.5b_{0.5}$. What is impact of this split strategy; in particular, is it more or less thant 0.5? Finally, same question for splitting between $a_{0.75}$ and $b_{0.75}$.

2. Next, consider the 2-group model using the values of the parameters and initial conditions you found from fitting each of your data sets in the previous assignment (set $p_{12} = 0$ if your fit yielded a negative value). The idea here is to compare type a intervention targeted primarily at group 1 versus type a intervention directed equally toward both groups. For the targeted strategy, set $b_1 = b_2 = 0$ and $a_1 = 10a_2$ and find how big you have to make (a_1, a_2) to have an impact of 0.5 and to have an impact of 0.75. Then, set $b_1 = b_2 = 0$ and $a_1 = a_2$ and see how big you have to make the a's to get the same impacts. Finally, determine the effects of the split strategies that average the two vectors $(a_1, a_2, 0, 0)$ of "impact 0.5" parameters you found and the two sets of "impact 0.75" parameters you found. Do these tasks for each of your two metropolitan areas.

3. Same question but for type b interventions.

4. Summarize and interpret your results.