AMSC/MATH 420, Spring 2015
Modeling Epidemics: Team Homework 2c
due Tuesday April 21

We'll continue with the two-group SI model with two types of interventions:

$$dS_1/dt = -p_{11}S_1\mathcal{I}_1 - p_{12}S_1\mathcal{I}_2 - a_1S_1$$
$$d\mathcal{I}_1/dt = p_{11}S_1\mathcal{I}_1 + p_{12}S_1\mathcal{I}_2 - (a_1 + b_1)\mathcal{I}_1$$
$$dS_2/dt = -p_{21}S_2\mathcal{I}_1 - p_{22}S_2\mathcal{I}_2 - a_2S_2$$
$$d\mathcal{I}_2/dt = p_{21}S_2\mathcal{I}_1 + p_{22}S_2\mathcal{I}_2 - (a_2 + b_2)\mathcal{I}_2.$$

We're modeling the cost of an intervention parameter quadruple (a_1, a_2, b_1, b_2) to be $K_c(a_1, a_2, b_1, b_2) = ca_1 + ca_2 + b_1 + b_2$, where c is a positive number. For this assignment, set a budget of $K_c(a_1, a_2, b_1, b_2) \leq 0.04$ and consider the optimal paramaters to be those within the budget that maximize the impact $M(a_1, a_2, b_1, b_2)$. (The budget value 0.04 should yield impacts that are somewhere between 0.1 and 0.99 for values of $c \geq 1$. If you are getting impacts that are very close to 0 or very close to 1 for your transmission parameters, adjust the budget to make the problem more interesting.)

For each of the two data sets assigned to your team, use the transmission parameters and initial conditions you found by fitting the two-group SI model (without interventions) to the data, and answer the following questions:

1. What is the largest value of c for which the optimal parameters have $b_1 = b_2 = 0$? (One decimal place of accuracy is fine for this and the next question.)

2. What is the smallest value of c for which the optimal parameters have $a_1 = a_2 = 0$?

3. For a range of c values in between the values you found above, determine the optimal a_1, a_2, b_1, b_2 and graph these values as a function of c. Are there values of c for which the optimal parameters are all positive, and/or for which 3 of 4 are positive?

Finally, discuss how you plan to address the questions raised in your multi-week project with each other and with the instructor. Write a paragraph that reflects the outcome of that discussion.